MODELLING OF TWO PHASE FLOW INSTABILITIES UNDER NATURAL CIRCULATION CONDITIONS USING NODAL TRANSMISSION MATRICES

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ABSTRACT

A rigourous and comprehensive model using the linearly perturbed drift flux form of conservation equations and the concept of nodal transmission matrices are developed to predict and explain stability characteristics and system responses under two phase natural circulation conditions in particular, and boiling-condensing flow in general, in a figure of eight geometry.

In this model, several important effects such as: vapour compressibility, wall heat storage capacity, nonequilibrium vapour generation, propagation delay, unequal velocity between the phases, and flow regime dependence through the relative drift velocity, are included in the formulation. For each of the regions of interest in the loop, a transmission matrix is constructed and this matrix determines how disturbances in flow, void fraction, pressure and enthalpy at the section inlet are propagated through that section. The overall characteristic transfer matrix of the system is a product of all the individual section transmission matrices. Stability features and system response can be determined from this characteristic matrix.

This paper gives an overview of the model philosophy, and formulations with the emphasis on the two phase region. Assumptions will be stated and justified. Essential physical features as the result of the formulation will be pointed out. Some results calculated by the model will be presented.

to be presented at the
12 th Simulation Symposium of Reactor Dynamics and Plant Control
McMaster University, Hamilton, Ontario

Introduction:

In recent years, considerable interest has been expressed concerning the phenomenon of two phase flow instability in two phase systems. The phenomenon of thermally induced, two phase flow instabilities is of interest for the design and operation of many industrial systems and equipment such as steam generators, thermosiphon reboilers, refrigeration plants and in particular the primary heat transport system of CANDU nuclear power plants. Oscillation of flow, and system pressure are considered to be undesirable as they can cause mechanical vibration, problems of system control, and in extreme circumtances it can disturb the heat transfer characteristics so that a heat transfer surface may burn out. In a nuclear power plant where burn out is to be avoided, flow oscillation can initiate transient dryout. Even before this extreme, sufficiently large oscillations in the normal operating state of the reactors can cause reactor trip setpoints to be exceeded, leading to frequent reactor trips. Large flow oscillations may also lead to early thermal fatigue, caused by continual cycling of the wall temperature. In the event of a LOCA, two phase flow instabilities may have adverse effect on natural circulation cooling scheme and stop the process needed to remove decay heat. It is clear from these problems that flow instabilities are to be avoided and every effort should be made to ensure that each plant has an adequate safety margin against them. Therefore, the understanding of two phase flow instabilities is extremely important to the design, control, performance of nuclear reactor power plant or any system having such a phenomenon. It is the objective of this study to look at this widely occuring phenomenon from a fundamental viewpoint. This paper will illustrate the theoretical approach of this study for a figure of eight geometry.

Overview of Model

A linear model has been formulated using the drift flux form of conservation equations (1), and the concept of transmission matrices (2,3) to determine the stability threshold and characteristics, system responses to disturbances in system parameters such as flow, void fraction, pressure, enthalpy, density etc. under natural and forced circulation conditions in a figure of eight loop. The model adopts the concept of the transmission matrix as a developmental base because of its inherent modularity (hence flexibility) and because its matrix structure is the necessary structure for efficient determination of system characteristics.

Consider the schematic figure of a simple figure of eight circuit which represents one complete pass in a CANDU primary heat transport system in figure 1. For each half of the circuit, it is divided into seven regions of interest according to their distinct conditions in wall heat flux, single or two phase etc. The first region (0-1) is the liquid line from one boiler outlet to the following heater inlet. This represents the inlet feeder pipe network in the PHTS. The second section (1-2) is the liquid line inside the heater where liquid is losing subcooling. This region ends at the boiling boundary. This section is distinguished from the former section because of the different heat fluxes in the two sections. The third section (2-3) is the two phase region inside the heater where major evaporation occurs. The fourth section (3-4) is the two phase line in the horizontal section of the hot leg piping. The fith section (4-5) is the two phase section in the vertical section of the hot leg piping. This division is deemed necessary even though wall heat loss is considered to be the same in the two sections because there is different vapour generation and condensation rate in these two sections. There will be some vapour generation due to elevation change in region 4-5 and under certain pressure conditions, there is a net vapour generation in this region. The sixth section (5-6) is the two phase section that penetrates past the boiler inlet. It is in this region that all remaining vapour will be condensed. The seventh section (6-7) is the liquid line in the boiler which starts at the condensing boundary and ends at the second boiler outlet. If symmetry is assumed, there will be seven identical regions for the second half. When these two halves are considered together, they represent the full figure of eight loop.

For each of the above sections, a transmission matrix or transfer matrix will be constructed. This transfer matrix will determine how disturbances in flow, void fraction, pressure and enthalpy at the section inlet are propagated through that section. An overall characteristic matrix of the system is a product of all the individual matrices. Because of symmetry, only half of the loop transfer matrix is needed and it is written as

$$G = \prod_{i=1}^{7} g_i$$

or in tensor notation as

$$\delta k_{out} = G_{ij}\, \delta k_{ji}$$

where k represents w, α, p, h and $G_{ij} = f$ (s, system conditions). The overall system characteristics matrix is thus,

$$\bar{\bar{H}} = \bar{\bar{I}} - \bar{\bar{G}} \cdot \bar{\bar{G}}.$$

The system characteristics can be determined from the zeroes of the determinant $|H|$. The zeroes are solved using Mueller's method of determining the roots of complex polynomials (4). The system will be unstable if any root has a positive real part or locates in the right half plane of the frequency domain (3). A positive real root(imaginary part=0) indicates instability of an excursive nature (Ledinegg instability), and a pair of complex roots with positive real part indicates oscillatory behaviour. In oscillatory flow, the appropriate phase of the flow, void fraction, pressure, enthalpy oscillation between the two halves of the loop are determined from the half loop transmission matrix.

Model Formulation:

With the exception of bubbly flow, the assumption of equal velocity between the two phases (like in homogeneous flow) is not very reasonable. However if the two phases are considered to be totally separated as in the two fluid model, the problem formulation may become very complex for an analytical model, and the uncer tainty of the types of constitutive equations that are to be used will not make prediction any more accurate. In between these two extremes, one method to take the relative velocity between the phases into account is the drift flux formulation.

In this model, the drift flux form of mixture mass, momentum and enthalpy conservation equations, along with the vapour phase continuity or void propagation equation, are used as the basic governing equations. However, these general equations can be further reduced for stability analysis (where we would only need to examine behaviour about a stable condition). As the model is derived for natural circulation flow (low velocity), the momentum diffusion and energy diffusion due to the relative motion between the phases can be neglected. Further all the covariances terms can be neglected. It is also assumed that the effect of the rate of change of individual phase density is neligible comparing to the rate of change of void fraction, i.e :

$$\frac{\partial \rho_f}{\partial t}, \frac{\partial \rho_g}{\partial t} \ll \frac{\partial \alpha}{\partial t}$$

This is also to say that the change in mixture density arises mainly from the change in void fraction. Because of the inherent modularity of the model approach, drift velocity can also be assumed to be constant in each section of interest. With these assumptions the one-dimensional governing equations are written as follows:

- Mixture continuity equation:

$$\Delta\rho \frac{\partial \alpha_g}{\partial t} - \frac{\partial G}{\partial z} = 0 \tag{1}$$

- Mixture momentum equation;

$$\frac{\partial G}{\partial t} + \frac{\partial}{\partial z}\left(\frac{G^2}{\rho_m}\right) = -\frac{\partial P_m}{\partial z} - \rho_m g \sin\psi - f_m \frac{G^2}{R\rho_m} \tag{2}$$

- Mixture enthalpy equation:

$$\rho_m \frac{\partial h_m}{\partial t} + G\frac{\partial h_m}{\partial z} = q'' \frac{\xi}{A} + \frac{\partial P_m}{\partial t} + \frac{G}{\rho_m}\frac{\partial P_m}{\partial z} \tag{3}$$

- Void propagation equation:

$$\frac{\partial \alpha_g}{\partial t} + \frac{\partial}{\partial z}(\alpha_g v_g) = \frac{\Gamma_g}{\rho_g} \qquad (4)$$

- Definition of drift flux velocity:

$$V_{gj} = v_g - C_o j \qquad (5)$$

where j is the volumetric flux and whose gradient is expressed as

$$\frac{\partial j}{\partial z} = \frac{\Gamma_g}{\rho_g}\left(\frac{\Delta\rho}{\rho_f}\right) - \frac{\alpha_g}{\rho_g}\left(\frac{\partial \rho_g}{\partial t} + v_g \frac{\partial \rho_g}{\partial z}\right) \qquad (6)$$

In equation (6), the first term represents the effect of vapour generation/condensation, the second term represents the compressibility effect of the vapour phase. The compressibility effect of the liquid phase is neglected.

The vapour velocity is defined as:

$$v_g = \frac{C_o G/\rho_f + V_{gj}}{\rho_f\left[1 - \Delta\rho C_o \alpha_g/\rho_f\right]} \qquad (7)$$

We introduce the dimensionless variables :

$$u_k = v_k / v_{ref} C_o \quad ; \quad u_{gj} = V_{gj}/v_{ref} C_o \quad ; \quad \theta = t/t_{ref} \quad ; \quad \zeta = z/L_c$$

$$\alpha = \alpha_g C_o \Delta\rho/\rho_f \quad ; \quad w = G/\rho_f v_{ref} \quad ; \quad P = P_m/\rho_f v_{ref}^2 \quad ; \quad h = h_m/h_{fg}$$

$$L_c = v_{ref} t_{ref} \quad ; \quad \rho = \rho_m/\rho_f = \left[\alpha_g \rho_g + (1-\alpha_g)\rho_f\right]/\rho_f$$

along with the following dimensionless grouping for each section:

- Phase change number $Q_2 = \dfrac{\Gamma_g \Delta\rho L_c C_o}{\rho_f \rho_g v_{ref}}$

- Subcooled number $Ja = (h_f - h_{fi})\rho_f v_{fg}/h_{fg}$

- Heat flux number $Q_1 = 2q'' L_c / R h_{fg} \rho_f v_{ref}$

- Froude number F_R $= \dfrac{\text{kinetic energy}}{\text{potential energy}} = v_{ref}^2/gL_c$

- Eckert number E_c $= \dfrac{\text{kinetic energy}}{\text{phase change energy}} = v_{ref}^2/h_{fg}$

- Friction factor $f = 2f_m L_c/R$

In natural circulation flow one can expect the Eckert number to be small (indeed it was calculated to be $10^{-6}-10^{-8}$ for our conditions), thus terms containing Eckert number can be eliminated.

The dimensionless governing equations are written as follows:

- Mixture mass conservation equation:

$$\frac{\partial \alpha}{\partial \theta} - \frac{\partial W}{\partial z} = 0 \tag{8}$$

- Mixture momentum conservation equation:

$$\frac{\partial W}{\partial \theta} + \frac{\partial}{\partial z}\left(\frac{W^2}{\rho}\right) = -\frac{\partial P}{\partial z} - \frac{\rho \sin \varphi}{F_R} - f\frac{W^2}{\rho}. \tag{9}$$

- Mixture enthalpy conservation equation:

$$\rho\frac{\partial h}{\partial \theta} + W\frac{\partial h}{\partial z} = Q_1 \tag{10}$$

- Void propagation equation:

$$\frac{\partial \alpha}{\partial \theta} + C_o u_g \frac{\partial \alpha}{\partial z} = Q_2(1-\alpha) + \alpha^2\left(\frac{\rho_f}{\rho_g \Delta\rho}\right)\left[\frac{\partial}{\partial \theta}\rho_g + C_o u_g \frac{\partial}{\partial z}\rho_g\right] \tag{11}$$

- Drift relation:

$$u_g = \frac{W + u_{gj}}{1-\alpha} \tag{12}$$

We introduce a linear perturbation in flow, void fraction, pressure, enthalpy, density etc. of the general form:

$$W = \bar{W} + \delta W^*(\theta, z) \; ; \quad \alpha = \bar{\alpha} + \delta^* \alpha(\theta, z)$$
$$P = \bar{P} + \delta \hat{P}^*(\theta, z) \; ; \quad h = \bar{h} + \delta h^*(\theta, z) \tag{13}$$

into equation (8)-(12), and neglect second and higher order terms. Then we Laplace transform the resulting perturbed equations into the frequency domain to obtain differential equation or set of differential equations in the spatial coordinate. the solution of these D.E. will yield the appropriate transmission matrix. Consideration is now given to each individual section.

Section 0-1 : (the liquid line)

The following assumptions are appropriate for this liquid line:

- Liquid phase is incompressible;
- Liquid density is constant in this section since heat loss is small in the inlet piping.

From appendix B, the transmission matrix for this section is

$$\begin{bmatrix} \delta w_1 \\ \delta \alpha_1 \\ \delta P_1 \\ \delta h_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -(s+2f\bar{w}/\rho_{01})L_{01}/L_c & 0 & 1 & 0 \\ 0 & 0 & 0 & \exp(-s\rho_{01}L_{01}/L_c\bar{w}) \end{bmatrix} \begin{bmatrix} \delta w_0 \\ \delta \alpha_0 \\ \delta P_0 \\ \delta h_0 \end{bmatrix} \tag{14}$$

Section 1-2 : (the subcooled liquid line)

The following assumptions are appropriate for this heated region :

- Liquid density varies linearly with axial position in the heater
- Heat storage capacity in the heater wall is accounted for using radially lumped analysis;
- Heat transfer coefficient to single phase liquid is obtained from the Dittus and Boeter correlation (6).

This results in the enthalpy equations for the wall and liquid as

- Wall :

$$\frac{2d_w}{R}\frac{\rho_w}{\rho_f}\frac{\partial h_w}{\partial \theta} = Q_1 - H_1\left(\frac{C_{pf}}{C_{pw}}h_w - h\right) \tag{15}$$

- Fluid :

$$\frac{\rho}{w}\frac{\partial h}{\partial \theta} + \frac{\partial h}{\partial z} = \frac{H_L}{w}\left(\frac{C_{pf}}{C_{pw}}h_w - h\right) \tag{16}$$

From appendix B, the resulting matrix is

$$\begin{bmatrix} \delta W_2 \\ \delta \alpha_2 \\ \delta P_2 \\ \delta h_2 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ B_{21} & 0 & 0 & B_{24} \\ B_{31} & 0 & 1 & 0 \\ B_{41} & 0 & 0 & B_{44} \end{bmatrix} \begin{bmatrix} \delta W_1 \\ \delta \alpha_1 \\ \delta P_1 \\ \delta h_1 \end{bmatrix} \qquad (17)$$

where

$$B_{21} = \frac{C_o \Delta\rho [C_o \rho_g + \rho_f \rho_g u_{gj} G_{uf} (\bar{x} + B_{41})}{[C_o(\bar{x}\Delta\rho + \rho_g) + \rho_f \rho_g u_{gj} G_{uf}]^2}$$

$$B_{24} = \frac{C_o \Delta\rho (C_o \rho_g + \rho_f \rho_g u_{gj} G_{uf}) B_{44}}{[C_o(\bar{x}\Delta\rho + \rho_g) + \rho_f \rho_g u_{gj} G_{uf}]^2}$$

$$B_{31} = -\frac{Ja\,\Delta\rho}{Q_1 \rho_f^2 v_{fg}} \left[s + 2f \frac{\rho_1}{(\rho_1 - \rho_f)} \ln\left(\frac{\rho_1}{\rho_f}\right) \right]$$

$$B_{41} = \frac{-Q_1(1 - B_{44})}{s[\frac{1}{2}(\rho_f + \rho_1)/\rho_f + (2H_1 d_w C_{pw} \rho_w / R C_{pf} \rho_f)/(H_1 + 2 s d_w C_w \rho_w / R C_p \rho_f)]}$$

$$B_{44} = \exp\left\{ \frac{-s Ja\,\Delta\rho}{Q_1 \rho_f^2 v_{fg}} \left[\frac{1}{2}(\rho_f + \rho_1)/\rho_f + (2 H_1 d_w C_{pw} \rho_w / R C_{pf} \rho_f)/(H_1 + 2 s d_w C_w \rho_w / R C_p \rho_f) \right] \right\}$$

General two phase line :

The approach in the formulation of the transmission matrix for all the two phase regions will be the same, so we can consider only one section in general and use the same set of equations for other sections with different dimensionless groups, conditions Several assumptions are made and they are stated in the following:

1. The change in void fraction is much more dominant than the change in the vapour density alone.
2. The liquid phase is incompressible. Only the vapour phase is compressible.
3. The pressure in the vapour and liquid phase is the same and is equal to the mixture pressure.
4. Due to low flow under natural circulation conditions, the accelerational component of pressure drop is neglected. Only the frictional and gravitational components are considered.
5. Thermodynamic equilibrium exists. The effect of thermodynamics nonequilibrium is taken care of by the use of constitutive relation (10).
6. Mixture density is related to mixture pressure and enthalpy

through state equation of the form $\rho = \rho\,(p,h)$. Thus,

$$\delta\rho_g = \left(\frac{\partial \rho_g}{\partial P}\right)_h \delta P + \left(\frac{\partial \rho_g}{\partial h}\right)_P \delta h$$

7. The effect of pressure on the mixture density is much more dominant than the effect of enthalpy. i.e

$$\delta\rho_g \simeq \left(\frac{\partial \rho_g}{\partial P}\right) \delta P$$

8. A dimensionless residence time of the form,

$$d\tau = \frac{1}{\bar{u}_g}\, dz \qquad (18)$$

is used to transform the axial coordinate to the residense time coordinate.

9. This transformation of variable along with steady state of (10) and (11) gives an expression for local residence time as

$$\tau = \frac{C_0}{Q_2} \ln\left[1 + \frac{Q_2}{C_0}\left(\frac{1-\bar{\alpha}_i}{\bar{w}+u_{gj}}\right) z \right] \qquad (19)$$

10. In the differential equation approach, the coefficients of the resulting perturbed and Laplace transformed D.E are linearized by averaging over the section. This will reduce the set of coupled D.E with variable coefficients to a more solvable set of D.E with constant coefficients. This assumption is justified as long as the system conditions do not deviate too far from the stable conditions i.e. near the stability threshold.

From appendix B, we have a sytem of 4 coupled D.E with variable coefficient, as follows:

- Perturbed void propagation equation with mixture continuity:

$$\frac{d^2\delta W}{d\tau^2} + \frac{sQ_2}{C_0}\delta W = -s^2\left(\frac{s+2Q_2}{C_0}\right)\left(\frac{\bar{w}+u_{gj}}{1-\bar{\alpha}_i}\right) e^{Q_2\tau/C_0}\,\delta\alpha$$
$$+ \frac{s^2 G_{uf}^2 \alpha^2}{\rho_g \Delta\rho C_0}\left(\frac{\partial\rho_g}{\partial P}\right)\left(\frac{\bar{w}+u_{gj}}{1-\bar{\alpha}_i}\right) e^{Q_2\tau/C_0}\,\delta P \qquad (20)$$

- Perturbed momentum equation:

$$\frac{d\delta P}{d\tau} = -\left(s + 2f/\bar{\rho}\right)\left(\frac{\bar{w}+u_{gj}}{1-\bar{\alpha}_i}\right) e^{Q_2\tau/C_0}\,\delta W - \frac{\Delta\rho}{\rho_f}\left(\frac{f}{\rho^2} - \frac{\sin\psi}{F_R}\right)\left(\frac{\bar{w}+u_{gj}}{1-\bar{\alpha}_i}\right)$$
$$e^{Q_2\tau/C_0}\,\delta\alpha \qquad (21)$$

- Perturbed mixture continuity:

$$\delta\alpha = \frac{1}{s}\left(\frac{1-\alpha_i}{\bar{w}+u_{gj}}\right) e^{-Q_2\tau/C_o} \frac{d\delta w}{d\tau} \qquad (22)$$

- Perturbed enthalpy equation:

$$\frac{d\delta h}{d\tau} + \left(s\bar{\rho} + \frac{2sH_2 d_w C_w \rho_w / RC_p\rho_f}{H + 2sd_w C_w \rho_w / RC_p \rho_f}\right)\left(\frac{\bar{w}+u_{gj}}{1-\bar{\alpha}_i}\right) e^{Q_2\tau/C_o}\,\delta h = Q_1\left(\frac{\bar{w}+u_{gj}}{1-\bar{\alpha}_i}\right) e^{Q_2\tau/C_o}\,\delta w \qquad (23)$$

Equation (20) has the form of the one dimensional, inhomogeneous, Helmholtz equation. This type of equation usually describes the space part of wave propagation, thus all the characteristics of wave propagation phenomenon will be applicable here. Therefore from the fundamental conservation equations, it is illustrated that the disturbances can be propagated in wave form. The two terms on the right hand side of equation (20) are the driving terms of the waves. The first term corresponds to the driving mechanism known as density wave and the second term represents the driving mechanism by compressibility.

From appendix B, the resulting matrix equation for each two phase region is written in the form:

$$\begin{bmatrix} \delta w_{out} \\ \delta\alpha_{out} \\ \delta P_{out} \\ \delta h_{out} \end{bmatrix} \begin{bmatrix} f_{11}(s,\tau_o) & f_{12}(s,\tau_o) & f_{13}(s,\tau_o) & 0 \\ f_{21}(s,\tau_o) & f_{22}(s,\tau_o) & f_{23}(s,\tau_o) & 0 \\ f_{31}(s,\tau_o) & f_{32}(s,\tau_o) & f_{33}(s,\tau_o) & 0 \\ f_{41}(s,\tau_o) & f_{42}(s,\tau_o) & f_{43}(s,\tau_o) & f_{44}(s,\tau_o) \end{bmatrix} \begin{bmatrix} \delta w_{in} \\ \delta\alpha_{in} \\ \delta P_{in} \\ \delta h_{in} \end{bmatrix} \qquad (24)$$

where τ_o is the dimensionless residence time in the section,

$$\tau_o = \frac{C_o}{Q_2} \ln\left[1 + \frac{Q_2}{C_o}\left(\frac{1-\bar{\alpha}_i}{\bar{w}+u_{gj}}\right) L_{sec}/L_c\right] \qquad (25)$$

and f_{ij} is a complicated function of system conditions.

Section 6-7 : (liquid line in steam generator)

The same assumptions as in region 1-2 are used in the formulation of the transmission matrix of region 6-7. The resulting matrix is as follows:

$$\begin{bmatrix} \delta w_7 \\ \delta \alpha_7 \\ \delta p_7 \\ \delta h_7 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ E_{31} & 0 & 1 & 0 \\ E_{41} & 0 & 0 & E_{44} \end{bmatrix} \begin{bmatrix} \delta w_6 \\ \delta \alpha_6 \\ \delta p_6 \\ \delta h_6 \end{bmatrix} \qquad (26)$$

where

$$E_{31} = -\left[s + 2\bar{f} \frac{1}{(\rho_o - \rho_f)} \ln(\rho_o/\rho_f) \right] L_{67}/L_c$$

$$E_{44} = \exp\left[-s\left(\frac{1}{2}\left(\frac{\rho_o + \rho_f}{\rho_f}\right) + \frac{2H_1 d_w C_{pw} \rho_w / R C_p \rho_f}{H_1 + 2s d_w C_{pw} \rho_w / R C_p \rho_f} \right) \frac{L_{67}}{L_c} \right]$$

$$E_{41} = \frac{Q_1 (1 - E_{44})}{s\left[\frac{1}{2}\left(\frac{\rho_o + \rho_f}{\rho_f}\right) + (2H_1 d_w C_{pw} \rho_w / R C_p \rho_f)/(H_1 + 2s d_w C_{pw} \rho_w / R C_{pf} \rho_f) \right]}$$

Steady State Calulation

-Boiling boundary:

From the steady state enthalpy equation for the wall and the liquid, assumming a direct transition from convection to single phase liquid to saturated boiling regime, the equilibrium sub-cooled boiling length can be shown to be:

$$\lambda_{eq} = \frac{GR}{2q''} \left(h_f - h_{fi} \right) \qquad (27)$$

- Void fraction distribution:

The steady state void propagation equation is

$$C_o \bar{u}_g \frac{\partial \bar{\alpha}}{\partial z} = Q_2 (1 - \bar{\alpha}) + \bar{\alpha}^2 \left(\frac{\rho_f}{\rho_g \Delta\rho} \right) C_o \bar{u}_g \frac{\partial \rho_g}{\partial z} \qquad (28)$$

In the second term on the right hand side, $\bar{\alpha}_{max}$ =1 and $\rho_f/\Delta\rho \geq 1$, i.e. $\bar{\alpha}^2 \rho_f/\rho_g \Delta\rho \lesssim 1$. We have already assumed that void fraction varies a lot more than vapour density, therefore, the second term on the RHS can be neglected. With the transformation of variable as in assumption (8), equation (28) becomes

$$\frac{\partial \bar{\alpha}}{\partial \tau} = \frac{Q_2}{C_o} (1 - \bar{\alpha}) \qquad (29)$$

Integrating equation (29) from its inlet, with inlet void fraction $\bar{\alpha}_i$, we have a void fraction distribution of the form:

$$(1-\bar{\alpha}) = (1-\bar{\alpha}_i)\ \exp\left(-\ Q_2 \tau / C_0\right) \qquad (30)$$

From equation (30), it can be deduced that void fraction increases along the section for $Q_2 > 0$ (net vapour generation), and decreases with $Q_2 < 0$ (net vapour condensation).

In the section with net vapour condensation, the location of total void condensation corresponds to the residence time for $\bar{\alpha} = 0$:

$$\tau_{cond} = \frac{C_0}{Q_2} \ln\left(1-\bar{\alpha}_i\right) \qquad (31)$$

The length of the two phase section up to the condensation point from the section inlet is determined by:

$$L_{cond} = L_c \frac{C_0}{Q_2}\left(\frac{\bar{w} + u_{gj}}{1-\bar{\alpha}_i}\right)\left[\exp\left(\tau_{cond}\frac{Q_2}{C_0}\right) - 1\right] \qquad (32)$$

By comparing this required length for condensation with the physical length of each section, it can be easily determined whether all void has condensed in that section. If the required length is longer than the section, then there is void penetration into the next section. The process is repeated until the location of total condensation is found.

Stability Consideration:

In performing the stability analysis, the distribution factor is calculated using the correlation (7),

$$C_0 = 1.2 - 0.2\sqrt{\rho_g/\rho_f}\ \left(1 - e^{-18\bar{\alpha}}\right) \qquad (33)$$

Experimental observation indicates three types of flow regime: slug flow, churn turbulent bubbly flow and annular flow. The transition between these flow regimes seems to follow the flow regime map by Weilsman (8). It was shown (5) that the drift velocity of the vapour depends upon the flow regime of the two phase mixture. For the first two types of flow regimes, the drift velocity does not depend upon void fraction (5) and can be simply calculated by:

-Churn turbulent flow

$$V_{gj} = 1.53\left[\sigma g \Delta\rho / \rho_f^2\right]^{1/4} \qquad (34)$$

-Slug flow

$$V_{gj} = 0.35 \left[g \Delta\rho / \rho_f \right]^{1/2}$$

For the annular flow pattern, more complicated expressions are required and they are fully documented by Ishii (7). The two phase friction factor is calculated using the single phase friction factor and a two phase friction multiplier coefficient. The single phase friction factor is calculated from the Blazius correlation (8) and the two phase friction multiplier coefficient is obtained from the Martinelli-Nelson correlation for separated flow (8).

The stability characteristics of the system are determined by the zeroes of the charateristics determinant $|\bar{\bar{H}}|$ where $\bar{\bar{H}} = \bar{\bar{I}} - \bar{\bar{G}}\bar{\bar{G}}$, and $\bar{\bar{G}}$ is the half loop transmission matrix. The system will be stable if all the zeroes of the determinant locate in the left hand side of the complex plane (roots with negative real part). The system will be unstable if any of the roots locate in the right plane of the complex domain (roots with positive real part) A complex root with a positive real part indicates oscillatory behaviour in the system.

Results:

Calculation were carried out at various heater powers, heater inlet temperatures, and loop inventory. Typical results are given here for heater power of 160 kw/channel and heater inlet temperature of 125 °C.

Figure 2 shows the calculated steady state pressure for header 1 at different loop inventories and primary mass flow rates. As loop inventory decreases i.e. increasing drainage, system pressure decreases as expected. As mass flow rate increases at the same inventory, pressure decreases as a result of lower temperature (better cooling) in the loop.

Figure 3 shows that Ja number which represents inlet subcooling decreases with increasing loop inventory and with increasing primary mass flow rate. This results directly from the reduction of system pressure for these variations.

Figures 4,5,6 and 7 show dimensionless vapour generation/condensation rate in the heater, horizontal hot leg piping, vertical hot leg piping, and the steam generator U tube, respectively. As loop inventory decreases, vapour content is expected to increase as is clearly shown in the above figures. It is also indicated in figure 5 and 6 that flashing always occurs in the vertical piping at this particular test condition and occurs in the horizontal piping for loop inventory less than 80%.

Figure 8 shows the net vapour generation rate which is the summation of all the Q_2 values of the tested section with a net vapour generation (positive Q_2 value). This generation component is expected to be one of the independent parameter that influences the stability of the system since this determines how much vapour is formed during the process.

Figure 9 shows approximately the model generated stability map at heater power of 160 kw/channel. A preliminary comparison between model prediction and experimental results shows reasonable quantitative agreement. A stable region is located in the region of high subcooling with low vapour generation and in a large part of the low subcooling region regardless of vapour generation. In the unstable region, both 180 out of phase and in-phase flow oscillation are predicted.

Conclusions:

An analytical model using the linearly perturbed drift flux form of the conservation equations and the concept of transmission matrices was developed to predict and explain the stability characteristics under natural circulation flow and boiling-condensing forced flow in a figure of eight geometry. The model includes several important effects such as vapour compressibility, wall heat storage capacity, nonequilibrium vapour generation/condensation, unequal velocity between the phases through the relative drift flux velocity, flow regime dependence.

Resulting equations have the form of the space part of the inhomogeneous wave propagation equation. The two driving mechanisms of the propagation result directly from the principle of conservation of mass and conservation of momentum.

Sample calculations by the model shows the expected trend of different parameters as a function of loop inventory. Although a direct comparison between the model's predictions and experimental results cannot be shown, quantitative agreement between the two results was found to be good.

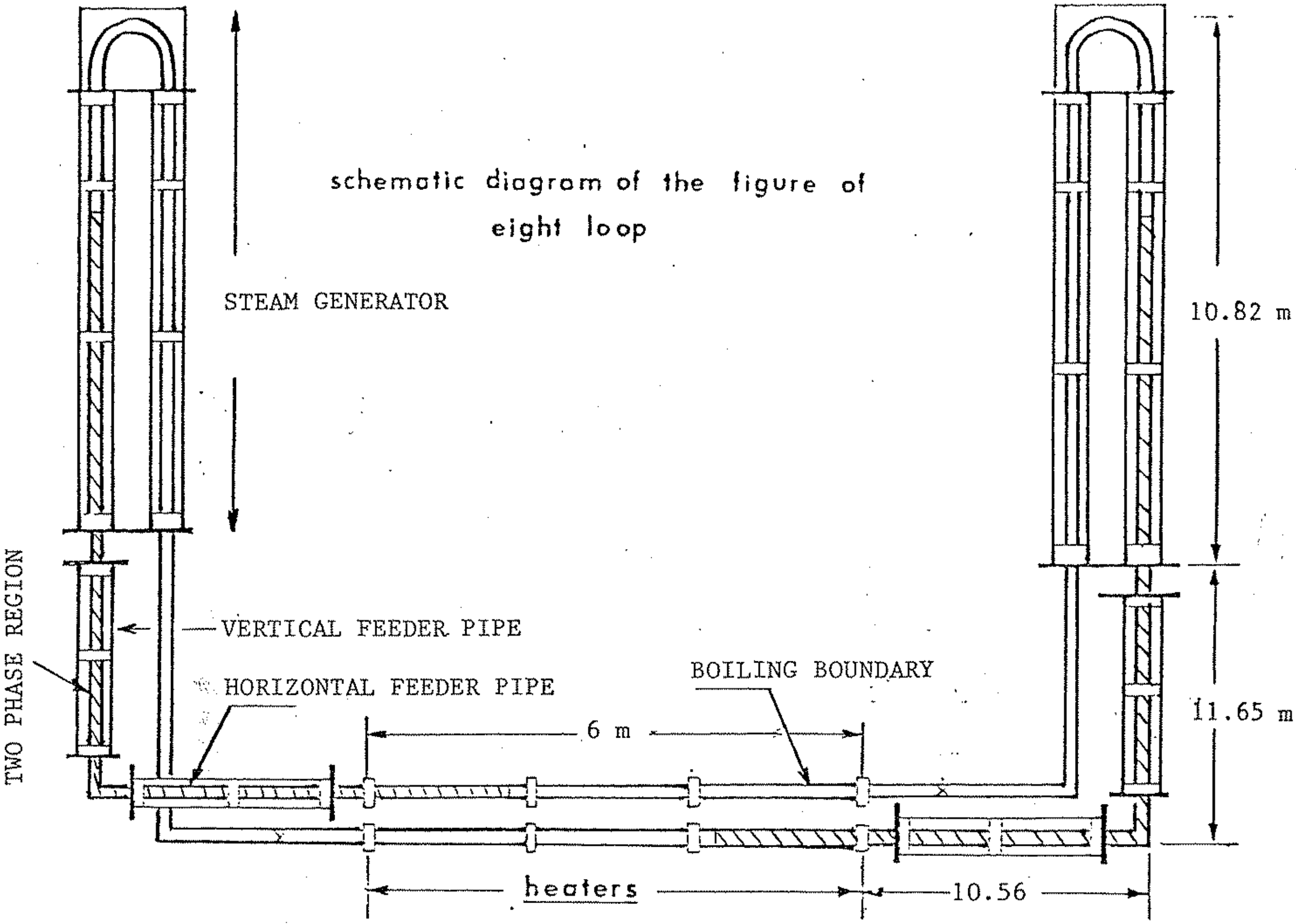


Fig.1 : Schematic diagram of the figure of eight loop

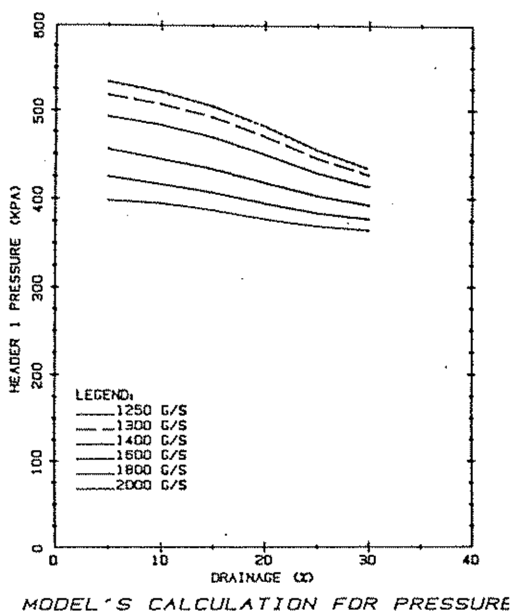


Fig 2 : Variation of Header 1 pressure at different loop inventory

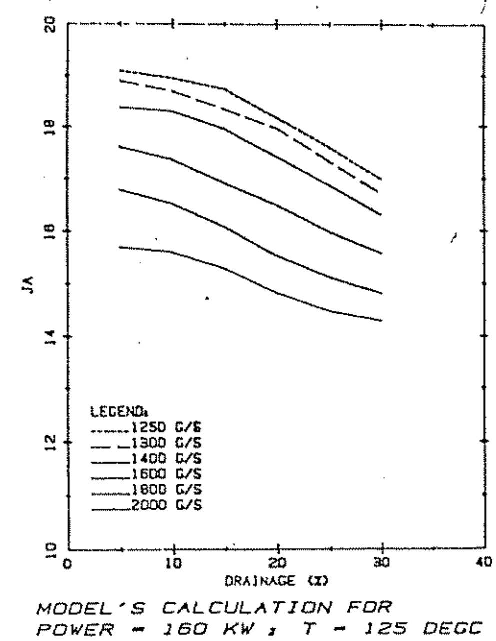


Fig 3 : Variation of Jacob number at different loop inventory

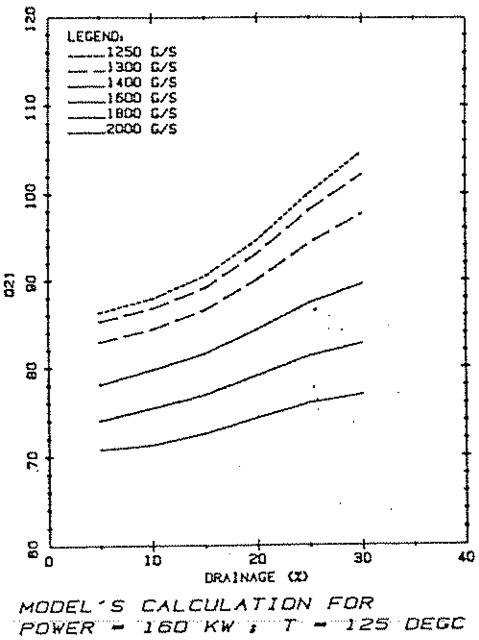


Fig 4 : dimensionless vapour generation rate inside heater at different loop inventory

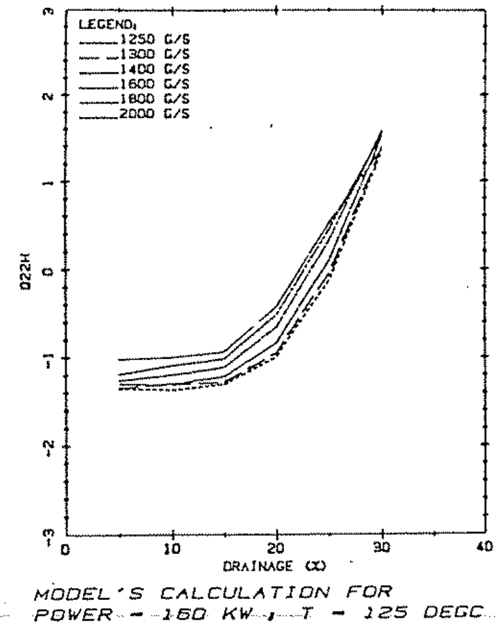


Fig 5 : Dimensionless vapour generation rate in the horizontal hot leg at different loop inventory

Fig 6 : Dimensionless vapour generation rate in the vertical hot leg at different loop inventory

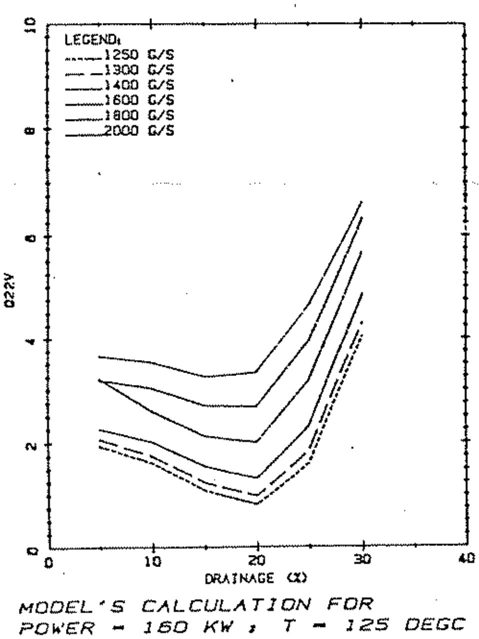
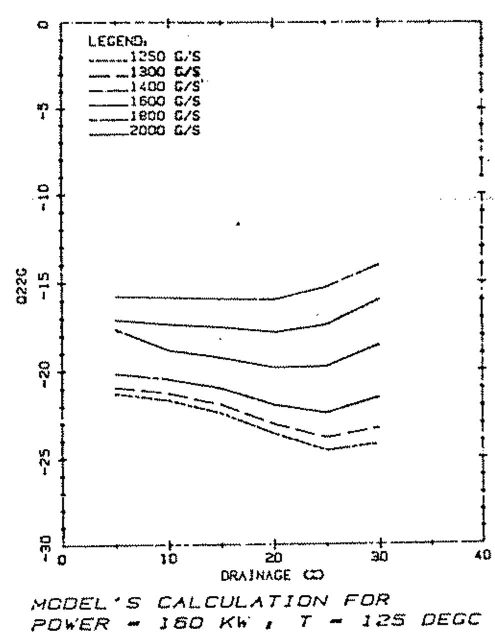


Fig 7 : Dimensionless vapour condens ion rate in the steam generator at different loop inventory



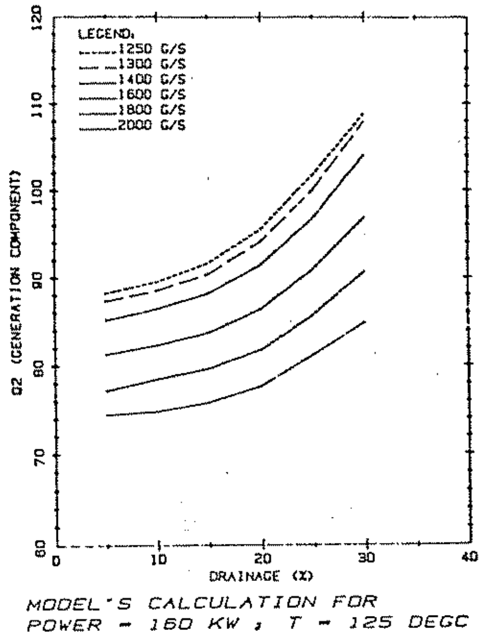


Fig 8 : Net vapour generation rate at different loop inventory

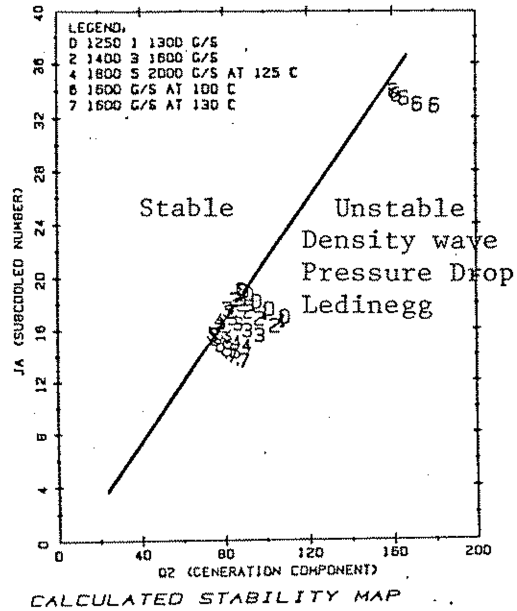


Fig.9 : Approximate stability map for large scale figure of eight loop

Nomenclature:

A : cross sectional area
C_p : heat capacity
D : pipe diamter
D_H : hydraulic diameter
Ec : Eckert number =
Fr : Froude number =
$\underline{G}$: mass flux
$\overline{G}$: half loop transmission matrix
H : dimensionless heat transfer coefficient
Ja : Jacob number (subcooling) = $(h_f - h_{fi}) \rho_f v_{fg} / h_{fg}$
K : heat conductivity
L_c : characteristics length
L_{ij} : length of section i-j
P_m : mixture pressure
Q_1 : dimensionless heat flux = $2q'' L_c / R h_{fg} \rho_f v_{ref}$
Q_2 : phase change number
R : radius of pipe
T : temperature
U : heat transfer coefficient
V_g : vapour velocity
V_{gj} : drift velocity
d_w : wall thickness of pipes
f_m : mixture friction factor
f : dimensionless friction factor =
g : gravitational acceleration constant
h_m : mixture enthalpy
h : dimensionless mixture enthalpy
h_w : dimensionless wall enthalpy
h_f : saturated liquid enthalpy
h_g : saturated vapour enthalpy
h_{fi} : liquid enthalpy at heater inlet
h_{fg} : = $h_g - h_f$
j : volumetric flux
p : dimensionless pressure
q'' : wall heat flux
s : Laplace variable
t : time
t_{ref} : reference time
u_g : dimensionless vapour velocity
u_{gj} : dimensionless drift velocity
v_g : $=1/\rho_g$, liquid specific volume
v_f : $=1/\rho_f$, vapour specific volume
:
v_{ref} : reference velocity
w : dimensionless mass flux
z : axial coordinate
x : thermodynamic quality

Greek:

α_g : dimensionless void fraction
α : void fraction

ξ : heated perimeter
Γ_g : equilibrium vapour generation
$\Gamma_{g,ne}$: non-equilibrium vapour generation
λ_{eq} : equilibrium boiling boundary
λ_{ne} : non-equilibrium boiling boundary
ρ : dimensionless mixture density
ρ_g : vapour density
ρ_f : liquid density
ρ_m : mixture density
θ : dimensionless time
τ : dimensionless residense time
ζ : dimensionelss axial coordinate
$\sin\psi$: pipe inclination

Subscript:

f : liquid phase
g : vapour phase
i : inlet
o : outlet
w : wall
ref: reference

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pendix A

Calculation of Vapour Generation Rate

Assuming a steady, low velocity, thermal equilibrium two phase flow in a uniform area pipe with elevation change, the energy equation and momentum equation can be written as:

$$G\frac{dh_m}{dz} = 2q''/R \qquad (A.1)$$

$$\frac{dP_m}{dz} = -\rho_m g \sin\psi - f_m \frac{\rho_m v_m^2}{R} - G\frac{dv_m}{dz} \qquad (A.2)$$

At thermodynamics equilibrium

$$h_m = x h_g + (1-x) h_f \qquad (A.3)$$

and where G, q'', $\sin\psi$, x, f_m denote the mixture mass flux, wall heat flux, pipe inclination, flow qualty, and friction factor respectively.
From (A.3),

$$\frac{dh_m}{dz} = h_{fg}\frac{dx}{dz} + \left[x\frac{dh_g}{dp} + (1-x)\frac{dh_f}{dp} \right]\frac{dP_m}{dz} \qquad (A.4)$$

At steady flow, we can neglect the accelerational pressure drop in the momentum equation. Using equation (A.1),(A.2) and (A.4) with the definition of vapour generation rate,

$$\Gamma_g = G\frac{dx}{dz}$$

we obtain the following equation for the vapour genration rate per unit volume:

$$\Gamma_g = \frac{2q''}{R h_{fg}} + \frac{G}{h_{fg}}\left[x\frac{dh_g}{dP_m} + (1-x)\frac{dh_f}{dP_m} - \frac{1}{\rho_m} \right]\left[\rho_m g \sin\psi + f_m \frac{G^2}{\rho_m R} \right] \qquad (A.5)$$

The first term in equation (A.5) represents the vapour generation/condensation due to heat addition/extraction. The heat extraction can be wall heat loss (like in the hot leg piping) or heat removal in the steam generator. The heat addition is the heater power input. The second term of equation (A.5) represents the vapour generation due to change in pressure of flashing. This flashing effect is significant in piping with a large pressure drop as in piping with elevation change.

Equation (A.5) can be written in non-dimensional form:

$$Q_2 = \Gamma_g \Delta\rho L_c C_o / \rho_g G_{ref} = Q_H + Q_F \qquad (A.6)$$

where $Q_H = \dfrac{2q''}{R h_{fg}} \left(\dfrac{\Delta\rho L_c C_o}{\rho_g G_{ref}} \right)$ = vapour generation or condensation due to heat addition or extraction

and

$$Q_F = \frac{1}{h_{fg}} \left[x \frac{dh_g}{dP} + (1-x) \frac{dh_f}{dP} - \frac{1}{\rho_m} \right] \left[\rho_m g \sin\psi + f_m \frac{G^2}{\rho_m R} \right] \left(\frac{\Delta\rho L_c C_o}{\rho_g} \right)$$

= vapour generation due to flashing.

To account for non-equilibrium effects in the drift flux equation, a constitutive relation obtained from a relaxation model by Saha and Zuber is used (10). It mainly introduces a correction factor to the thermal equilibrium vapour generation rate. This correction factor takes into account that the boiling boundary starts where significant vapour generation starts.
The thermal non-equilibrium vapour generation rate is written as:

$$\Gamma_{g,noneq} = \Gamma_{g,eq} \frac{1}{(L-\lambda)} \int_{\lambda}^{L} \left[1 - \exp\left(\frac{-z-\lambda}{\lambda_{eq}-\lambda} \right) \right] dz \qquad (A.7)$$

where

$$\Delta h_\lambda = 0.0022\, q'' D_h C_p / k_f \qquad \text{if } Pe = \frac{G D_h C_p}{k_f} \leq 70{,}000$$

or $\qquad (A.8)$

$$\Delta h_\lambda = 154\, q'' / G \qquad \text{if } Pe > 70{,}000$$

The parameter λ denotes the boiling boundary where significant vapour generation starts and the parameter λ_{eq} denotes the thermal equilibrium boiling boundary.

$$\text{For } \Delta h_{sub} > \Delta h_\lambda \qquad \lambda = \frac{GA(\Delta h_{sub} - \Delta h_\lambda)}{q'' \xi} \qquad (A.9)$$

$$\text{For } \Delta h_{sub} \leq \Delta h_\lambda \qquad \lambda = 0$$

Appendix B

Schematic of The Model Formulation

Dimensionless governing conservation equations for two phase flow mixture are written as follows:

- Mixture continuity equation:

$$\frac{\partial \alpha}{\partial \theta} - \frac{\partial W}{\partial z} = 0 \qquad (B.1)$$

- Mixture Momentum equation:

$$\frac{\partial W}{\partial \theta} + \frac{\partial}{\partial z}\left(\frac{W^2}{\rho}\right) = -\frac{\partial P}{\partial z} - \frac{\rho \sin\psi}{F_R} - f\frac{W^2}{\rho} \qquad (B.2)$$

- Mixture enthalpy equation:

$$\frac{\rho}{W}\frac{\partial h}{\partial \theta} + \frac{\partial h}{\partial z} = \frac{Q_1}{W} \qquad (B.3)$$

- Void propagation equation:

$$\frac{\partial \alpha}{\partial \theta} + C_o u_g \frac{\partial \alpha}{\partial z} = Q_2(1-\alpha) + \alpha^2\left(\frac{\rho_f}{\rho_g \Delta\rho}\right)\left[\frac{\partial \rho_g}{\partial \theta} + C_o u_g \frac{\partial \rho_g}{\partial z}\right] \qquad (B.4)$$

- Drift relation:

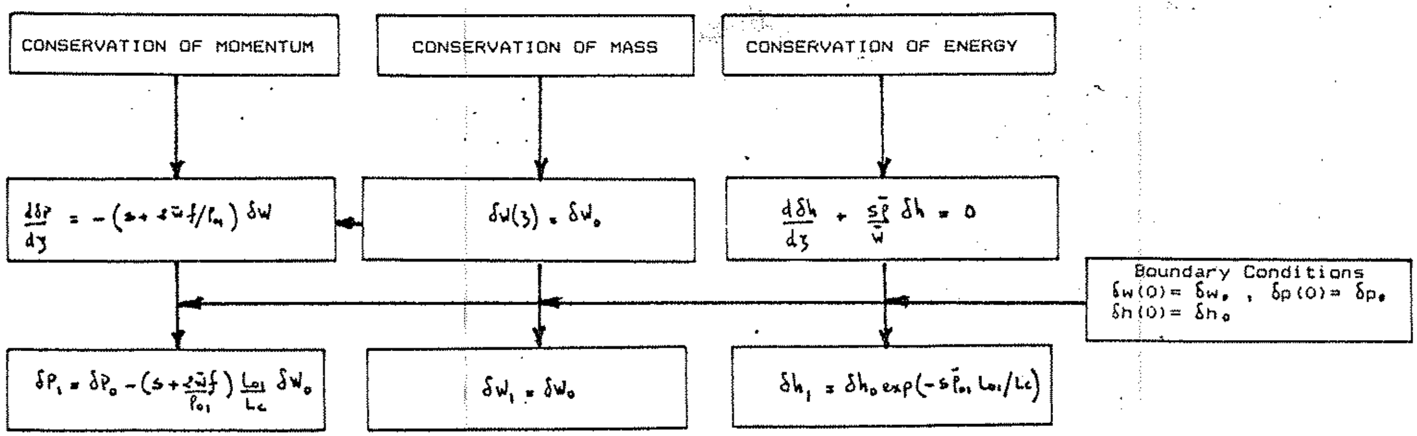
$$u_g = \frac{W + u_{gj}}{1-\alpha} \qquad (B.5)$$

- State equation:

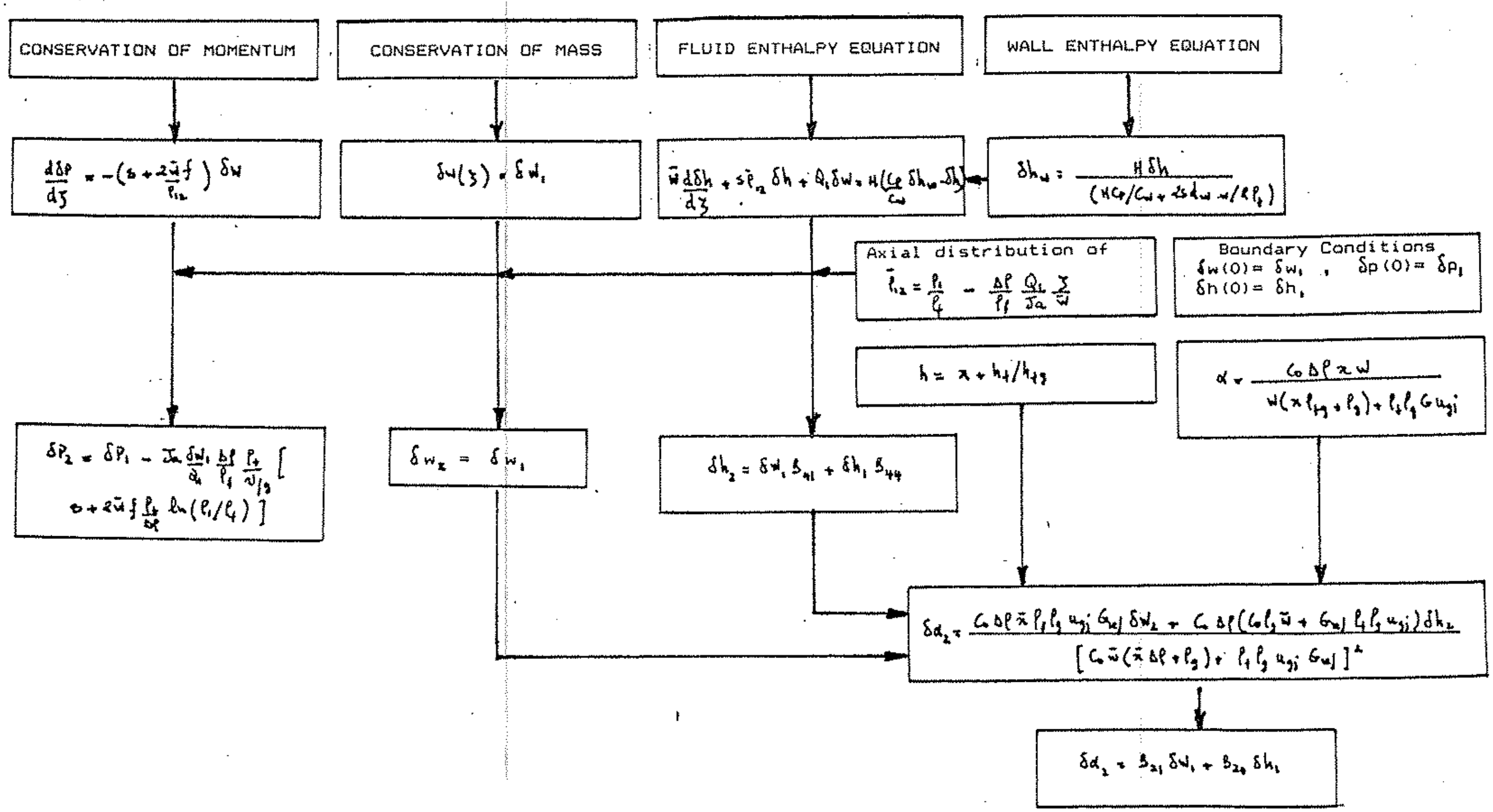
$$\rho = \rho(P, h)$$

$$d\rho = \left(\frac{\partial \rho}{\partial P}\right)_h dP + \left(\frac{\partial \rho}{\partial h}\right)_P dh \qquad (B.6)$$

SF ION 0-I : The liquid line



SECTION I-2 : The liquid line inside heater (subcooled region)



TWO PHASE LINE

Drift relation
$$u_g = \frac{w + u_{gj}}{1-\alpha}$$

$$\delta u_g = \frac{\delta w + \bar{u}_g\,\delta\alpha}{(1-\bar{\alpha})}$$

Equation of state
$$\rho = \rho(P, h)$$

$$\delta\rho_g = \left(\frac{\partial\rho_g}{\partial P}\right)\frac{G_{wj}^2}{\rho_f}\,\delta P + \frac{1}{h_{fg}}\left(\frac{\partial\rho_g}{\partial h}\right)\delta h_g$$

$$d\tau = \frac{1}{\bar{u}_g}\,dz$$

Steady state void propagation equat.

$$(1-\bar{\alpha}) = (1-\bar{\alpha}_i)\,e^{-Q_1\tau/C_0}$$

Void Progation Equation

$$\frac{d^2\delta w}{d\tau^2} + \frac{sQ_1}{C_0}\,\delta w = -s^2\left(\frac{s+2Q_1}{C_0}\right)\left(\frac{\bar{w}+u_{gj}}{1-\bar{\alpha}_i}\right)e^{Q_1\tau/C_0}\,\delta\alpha + \frac{s^2 G_{wj}^2\alpha^2}{\rho_g\,\Delta\rho\,C_0}\left(\frac{\partial\rho_g}{\partial P}\right)\left(\frac{\bar{w}+u_{gj}}{1-\bar{\alpha}_i}\right)e^{Q_1\tau/C_0}\,\delta P \qquad (B.7)$$

Mixture density

$$\delta\rho = -\frac{\Delta\rho}{\rho_f}\,\delta\alpha + \frac{\bar{\alpha}}{\rho_f}\,\delta\rho_f$$

$$\delta u_g = (\delta w + \bar{u}_g\,\delta\alpha)/(1-\bar{\alpha})$$
$$\delta\rho_g = \left(\frac{\partial\rho_g}{\partial P}\right)\frac{G_{wj}^2}{\rho_f}\,\delta P$$
$$d\tau = dz/\bar{u}_g$$
$$(1-\bar{\alpha}) = (1-\bar{\alpha}_i)\,e^{-Q_1\tau/C_0}$$

Momentum equation

$$\frac{d\delta P}{d\tau} = -\left(s + \frac{2f}{\rho}\right)\left(\frac{\bar{w}+u_{gj}}{1-\bar{\alpha}_i}\right)e^{Q_1\tau/C_0}\,\delta w - \frac{\Delta\rho}{\rho_f}\left(\frac{f}{\bar{\rho}^2} + \frac{\sin\psi}{Fr}\right)\left(\frac{\bar{w}+u_{gj}}{1-\bar{\alpha}_i}\right)e^{Q_1\tau/C_0}\,\delta\alpha \qquad (B.8)$$

$$\delta u_g = (\delta w + \bar{u}_g\,\delta\alpha)/(1-\bar{\alpha})$$
$$\delta\rho_g = \left(\frac{\partial\rho_g}{\partial P}\right)\frac{G_{wj}^2}{\rho_f}\,\delta P$$
$$d\tau = dz/\bar{u}_g$$
$$(1-\bar{\alpha}) = (1-\bar{\alpha}_i)\,e^{-Q_1\tau/C_0}$$

Enthalpy equation

$$\frac{d\delta h}{d\tau} + \left(s\bar{\rho} + \frac{2s d_w C_w \rho_w H/RC_p\rho_f}{H + 2s d_w C_w \rho_w/RC_p\rho_f}\right)\left(\frac{\bar{w}+u_{gj}}{1-\bar{\alpha}_i}\right)e^{Q_1\tau/C_0}\,\delta h = Q_1\left(\frac{\bar{w}+u_{gj}}{1-\bar{\alpha}_i}\right)e^{Q_1\tau/C_0}\,\delta w \qquad (B.9)$$

Equation (B.7), (B.8) , (B.9)

Average the coefficient to reduce to D.E. with constant coeffeciențs

System of D.E

$$(D^2 + a_4 D + a_5)\,\delta w - a_6\,\delta p = 0$$
$$D\delta p + (a_3 D + a_2)\,\delta w = 0$$
$$(D + a_1)\,\delta h - a_7\,\delta w = 0$$

Transformation of variable to residense time coordinates

Solution method for solving D.E

Boundary Conditions
$\delta w(0) = \delta w_i$, $\delta\alpha(0) = \delta\alpha_i$
$\delta p(0) = \delta p_i$, $\delta h(0) = \delta h_i$

Residense time at the section outlet

$$\delta w_{out} = f_{11}(s,\tau_0)\,\delta w_i + f_{12}(s,\tau_0)\,\delta\alpha_i + f_{13}(s,\tau_0)\,\delta P_i$$

$$\delta\alpha_{out} = f_{21}(s,\tau_0)\,\delta w_i + f_{22}(s,\tau_0)\,\delta\alpha_i + f_{23}(s,\tau_0)\,\delta P_i$$

$$\delta P_{out} = f_{31}(s,\tau_0)\,\delta w_i + f_{32}(s,\tau_0)\,\delta\alpha_i + f_{33}(s,\tau_0)\,\delta P_i$$

$$\delta h_{out} = f_{41}(s,\tau_0)\,\delta w_i + f_{42}(s,\tau_0)\,\delta\alpha_i + f_{43}(s,\tau_0)\,\delta P_i + f_{44}(s,\tau_0)\,\delta h_i$$