CV·ref C17

# DEVELOPMENT AND APPLICATIONS OF IDRIFF TWO-PHASE SIMULATION CODE

R. Sollychin, W.J. Garland and J.S. Chang
Department of Engineering Physics
McMaster University
Hamilton, Ontario

## Abstract

The IDRIFF (Integrated Drift-flux Formulation) simulation code has recently been developed as a convenient tool designed for the systematic and detailed analyses of two-phase flow. It can also be used to verify or to supplement existing thermal-hydraulic system codes. The IDRIFF code has two major model components : (1) lumped formulation for the fluid conservation equations and the equation of state, and (2) one-dimensional drift-flux formulation, each of which consists of five differential equations: mixture continuity, mixture momentum, mixture energy, dispersed phase continuity and equation of state. The IDRIFF is an interactive code providing many flexible control features to suit most of the needs in two-phase simulation applications.

The first of such applications has been carried out in a study of a pressurizer. The special version of the IDRIFF code for the study is described in this paper. Preliminary results of the application are also discussed.

## 1.0 General Description Of The IDRIFF Code

The main objective of developing the IDRIFF (Integrated Drift-flux Formulation) code is to provide a flexible tool in the analysis of two-phase flow which occasionally requires detailed information on local phenomena in the flow. The basic structure of the code is a set of fluid mass, momentum and energy balance equations and the equation of state, written in lumped (integrated) form. The behaviour of a two-phase flow system can be simulated by dividing the system into a structure of control volumes (similar to the nodes used in the existing homogeneous system codes such as SOPHT or FIREBIRD). The lumped formulation is then applied to each of the control volumes in a way similar to the existing system codes. However, the IDRIFF code provides an additional unique and powerful feature: when there is a need to obtain detailed profiles of flow parameters in any one or more of the control volumes, the lumped formulation on the control volume(s) can be replaced by sets of one-dimensional drift-flux model formulations. The drift-flux formulation consists of five differential equations : mixture continuity,mixture momentum, mixture energy, dispersed phase continuity and equation of state. Morover, it is believed that with little modification, such as adopting the nodalization structure of an existing system code into the IDRIFF code and setting-up a data communication mechanism between the IDRIFF code and a system code, the IDRIFF code can also be used to supplement or verify the system code.

---

For presentation at the 12th Simulation Symposium On Reactor Dynamics And Plant Control, Hamilton, Ontario, April 1986.

The IDRIFF is an interactive, highly flexible computer code currently residing in the McMaster S.S. VAX 11/780. Before and during a simulation, each control volume is divided into grid-points, the number of which is dynamically determined by the size of the control volume and by the size of the increment between grid-points. The latter is pre-set by the user for an individual control volume. When the drift-flux formulation is used in the calculation of a control volume, the program time-step is divided into sub-time-steps, the number of which is user controlled and is applicable only to that particular control volume. At the end of each time-step, upon a signal from the keyboard, access to the program run-control is available to the user, so that choices on the type of model (lumped or drift-flux), divisions of time-step into sub-time-steps, and type of output (screen display, storing into files, hard-copy or any combinations of the above) can be made by the user.

The organization of the two-phase system into control volumes is based on the system's structure and flow-regimes in such a way that each control volume is dominated by only one type of flow-regime. When a detailed calculation is required, the lumped formulation is substituted by drift-flux formulation which is appropriate for that flow-regime. During the simulation, the size of all the control volumes are dynamically adjusted according to the boundary of the flow-regimes as well as to the calculation of swelling or shrinking in the control volumes which are dominated by the liquid phase. This will be described in more detail in the Section 4.2.

When the drift-flux formulation is used, values of local drift-flux parameters are integrated over the control volume and their volume-averaged values are determined. The purpose of obtaining these averaged values is two-fold. First, averaging ensures that the drift-flux formulation is compatible with the rest of the IDRIFF models' structure, which is based on the lumped formulation. Second, the averaged values are used to setup boundary conditions for neigboring control volumes. The values of quality based on averaged specific enthalpy and averaged pressure is used in the calculation of swelling or shrinking of the control volume. Based on the new volume, the grid-points are reorganized at each time step and the values of the drift-flux parameters over the new grid-points structure in the control volume are distributed.

## 2.0 Special Features of IDRIFF Code for Pressurizer Analysis

Version 2 of the IDRIFF code is specially designed for the analysis of pressurizer behaviour. The pressurizer is divided into only two control volumes, with the one dominated by the gas phase geometrically above the liquid phase dominated one. From experimental observation, it is found that the possible flow-regimes in the bottom control volume are single-phase liquid, bubbly flow, froth flow and bubbly/froth mixture. The flow-regime in the top control volume is either single-phase vapour or droplet flow. For both volumes, therefore, only one type of drift-flux formulation is required, that is, one suitable for dispersed flow. For numerical convenience, however, two sets of this type of drift-flux formulation are prepared, one for the flow-regime in which the liquid phase is the continum phase and one for the flow-regime in which the gas phase is the continum phase.

In the lumped formulation, the momentum equation is simplified. It is only used to calculate the vertical movement of the liquid surface due to pressure difference between the steam (top) control volume and the liquid (bottom) control volume, rather than to be used as one of the governing equations. The following are added to the IDRIFF code : bubble-rise submodel in the liquid control volume, droplet-drop submodel in the steam control volume, interface evaporation and condensation submodel on the liquid surface, steam-bleed flow submodel and surge-line flow submodel. Furthermore, submodels on heat generation from electrical heaters, distribution of heat due to interface evaporation and condensation, and heat loss/gain due to steam-bleed and surge flow are also implemented.

Version 2 of the IDRIFF code has been tested. The procedure and its results are presented in section 5. The rest of this paper will be devoted to this particular version of the code.

## 3.0 Description Of Basic Physical Models

### 3.1 Definitions And Basic Assumptions

The underlying principle adopted in the IDRIFF model is the assumption of mechanical as well as thermal non-equilibrium. In the lumped formulation, there is a non-equilibrium between the two control volumes. From the modeling viewpoint, the average pressure, temperature and other thermodynamic properties in each of the control volume are determined independently of each control volume.

Through the process of mass and energy exchange between the control volumes and their common environment, such as steam-bleed, surge flow and heat generation from the heaters, and of mass and energy exchange between the two control volumes themselves, each of the control volume continuously possesses a new set of thermodynamic properties. In terms of extensive properties, they are the total mass $M_S$ and $M_L$, the total enthalpy $H_S$ and $H_L$, and the total volume $V_S$ and $V_L$ in the steam control volume and liquid control volume respectively.

The average pressure in the steam volume $P_{SA}$ is determined from the steam control volume average specific enthalpy $h_S$ (defined as $H_S/M_S$) and average density $\rho_S$ (defined as $M_S/V_S$). Similarly, the average pressure in the liquid control volume $P_{LA}$ is also determined from the liquid control volume's average specific enthalpy $h_L$ and average density $\rho_L$. This will be discussed further in the Section 4.1

The steam control volume and the liquid control volume are assumed to be at each of their own saturation condition under $P_{SA}$ and $P_{LA}$ respectively. Values of their average temperatures and the rest of their thermodynamic properties are therefore functions of their respective average pressure and their respective qualities $x_L$ and $x_S$.

In the drift-flux formulation, non-equilibrium between the two phases exists locally at each grid-point. It is worth noting that in the liquid control volume, the gas phase is the dispersed phase and the liquid phase is the continum phase. The reverse is true for the steam control volume. Furthermore, some mathematical operations used in the expressions of drift-flux parameters are needed to be defined before further discussions :

An area averaged over the pressurizer cross-sectional area A is given by:

$$<F> = \frac{1}{A} \int_A F \, dA$$

and the phase volume-fraction weighted mean value is:

$$<<F_k>> = \frac{<\alpha_k F_k>}{<\alpha_k>}$$

where k indicates the $k^{th}$ phase. It can be used to indicate the phase directly such as in local gas phase pressure $P_g$ and local liquid phase pressure $P_l$. Or it can be used to indicate the role of the phase in the flow-regime such as the local dispersed phase pressure $P_d$ and the local continuum phase pressure $P_c$; and $\alpha_k$ is the $k^{th}$ phase volume fraction (eg. $\alpha_d$ is the void fraction in the bubbly flow)

The local values of pressure, density and specific enthalpy of each phase within any cross-sectional area is assumed to be uniform, so that

$$P_k = <<P_k>>$$

$$\rho_k = <<\rho_k>>$$

$$h_k = <<h_k>>$$

The local area averaged mixture pressure $<P_m>$, which is defined as:

$$<P_m> = <\alpha_d> P_d + (1 - <\alpha_d>) P_c \tag{3.1}$$

is assumed to be the saturation pressure of the mixture at the local grid point. The values of $P_d$ and $P_c$ at each grid-point are determined through eqn.(3.1) as well as the following :

$$P_d = P_c + \Delta P \tag{3.2}$$

where $\Delta P$ is the local interphase pressure difference due to outer surface tension of the dispersed particles such as bubbles and droplets:

$$\Delta P = \text{function (particle size, } T_i)$$

where $T_i$ is the local interface temperature and is assumed to be equal to the local dispersed phase temperature $T_d$, i.e.:

$T_i = T_g$ in liquid control volume

and $T_i = T_l$ in steam control volume

Moreover, the dispersed phase is assumed to be at saturation locally with respect to the local dispersed phase pressure. For example, the dispersed phase temperature is calculated as :

$T_g = T_{sat}(P_g)$ in liquid control volume

and

$T_l = T_{sat}(P_l)$ in steam control volume.

On the other hand, the continuum phase is allowed to depart from its saturation state locally with respect to the local continuum phase pressure. Hence in the liquid control volume, the liquid phase can be either at saturation or subcooled state with respect to liquid phase pressure.

$T_l$ and other liquid-phase properties = functions ($P_l$, $h_l$) (in liquid c.v.)

while in the steam control volume, the gas phase is either at saturation or superheated state with respect to gas phase pressure.

$T_g$ and other steam phase properties = functions ($P_g$, $h_g$) (in steam c.v.).

Other useful definitions are the local averaged mixture density:

$$<\rho_m> = <\alpha_d> \rho_d + (1 - <\alpha_d>) \rho_c \tag{3.3}$$

and the local mixture velocity :

$$\bar{v}_m = \frac{[<\alpha_d> \rho_d <<v_d>> + (1 - <\alpha_d>) \rho_c <<v_c>>]}{<\rho_m>} \tag{3.4}$$

where $v_d$ and $v_c$ are the local velocity of the dispersed phase and continuum phase respectively.

The local mean mixture specific enthalpy is

$$\bar{h}_m = \frac{[<\alpha_d> \rho_d <<h_d>> + (1 - <\alpha_d>) \rho_c <<h_c>>]}{<\rho_m>} \tag{3.5}$$

In the process of integrating values of drift-flux parameters over the control volume and converting them into the corresponding lumped parameters, the following are assumed : The averaged pressure is set to be equal to the volume average of the local mixture pressure which is mathematically equivalent to the volume average of the area average mixture pressure. Hence

$$P_{SA} = \frac{1}{V_S} \int_{V_S} <P_m> dV \tag{3.6a}$$

$$P_{LA} = \frac{1}{V_L} \int_{V_L} <P_m> dV . \tag{3.6b}$$

The averaged specific enthalpy are calculated from mass averaged values of the local mean mixture specific enthalpy :

$$h_L = \frac{\int_{V_L} \bar{h}_m <\rho_m> dV}{\int_{V_L} <\rho_m> dV} \tag{3.7a}$$

and

$$h_S = \frac{\int_{V_S} \bar{h}_m <\rho_m> dV}{\int_{V_S} <\rho_m> dV} \tag{3.7b}$$

Similar to eqn. (3.6), the averaged density is calculated by volume averaging the area averaged mixture density

$$\rho_S = \frac{1}{V_S} \int_{V_S} <\rho_m> dV \tag{3.8a}$$

and

$$\rho_L = \frac{1}{V_L} \int_{V_L} <\rho_m> dV \tag{3.8b}$$

And the bubbles rise velocity $W_{BR}$ and the droplets drop velocity $W_{CD}$ can be calculated from local dispersed phase velocities :

$$W_{BR} = \frac{1}{V_L} \int_{V_L} <<v_d>> <\alpha_d> A \rho_d dV \tag{3.9}$$

and

$$W_{CD} = \frac{-1}{V_S} \int_{V_S} <<v_d>> <\alpha_d> A \rho_d dV . \tag{3.10}$$

Finally, an important assumption is that the liquid (bottom) control volume, which is dominated by liquid phase, is much more incompressible than the steam (top) control volume, which is dominated by the gas phase. Hence the calculation of swelling and shrinking is only done for the liquid control volume, and the volume in the steam control volume will satisfy :

$$\frac{dV_S}{dt} = - \frac{dV_L}{dt} \tag{3.11}$$

## 3.2 Lumped Formulation

The rate of change of mass in the control volumes can be expressed by the following :

$$\frac{dM_S}{dt} = -W_{STB} - W_{CD} - W_{CI} + W_{EI} + W_{BR} \tag{3.12}$$

$$\frac{dM_L}{dt} = W_{SRL} - W_{EI} - W_{BR} + W_{CD} + W_{CI} , \tag{3.13}$$

where $W_{STB}$ is the steam bleed flow, $W_{SRL}$ is the surge-line flow, $W_{CI}$ is the interface condensation rate at the liquid surface separating the steam control volume from the liquid control volume, $W_{EI}$ is the interface evaporation rate at the same liquid surface, $W_{CD}$ is the flow of condensate droplets (liquid phase) from the bulk of the steam control volume toward the liquid control volume, and $W_{BR}$ is the rising flow of bubbles (gas phase) from the bulk of liquid volume toward the steam volume.

The rate of change of energy in the two control volumes can be expressed by the following :

$$\frac{dH_S}{dt} = -W_{STB} \cdot h_{gST} - W_{CD} \cdot h_{fST} - W_{CI} \cdot h_{gST} + W_{EI} \cdot h_{gLQ}$$

$$+ W_{BR} \cdot h_{gLQ} - Q_{WS} + Q_{TR} - (1-\beta)[(1-\delta)Q_{COND} + Q_{EVPR}] \tag{3.14}$$

and

$$\frac{dH_L}{dt} = -W_{SRL} \cdot h_{SRL} - W_{EI} \cdot h_{fLQ} - W_{BR} \cdot h_{gLQ} + W_{CI} \cdot h_{fST}$$

$$+ W_{CD} \cdot h_{fST} - Q_{WL} + Q_{PWR} - Q_{TR} - \beta[(1-\delta)Q_{COND} + Q_{EVPR}], \tag{3.15}$$

where $h_{SRL}$ is the specific enthalpy of the fluid in the surge-line, $h_{gST}$ and $h_{fST}$ are respectively the saturated gas phase specific enthalpy and the saturated liquid phase specific enthalpy in the steam control volume, $h_{gLQ}$ and $h_{fLQ}$ are respectively the saturated gas phase specific enthalpy and the saturated liquid phase specific enthalpy in the liquid control volume, $Q_{WS}$ and $Q_{WL}$ are the rate of heat loss to the wall in the steam control volume and in the liquid control volume respectively, $Q_{TR}$ is the heat transfer rate from the liquid control volume to the steam control volume due to any temperature gradient, excluding those due to interface evaporation and condensation; $Q_{COND}$ is the rate of energy released by the condensing steam during the interface condensation process

$$Q_{COND} = W_{CI}(h_{fST} - h_{gST}), \tag{3.16}$$

and $Q_{EVPR}$ is rate of energy absorbed by the evaporating liquid during the interface evaporation process

$$Q_{EVPR} = W_{EI}(h_{gLQ} - h_{fLQ}). \tag{3.17}$$

The ratio, $\beta$, represents the portion of energy released during interface condensation or the portion of energy absorbed during interface evaporation contributed by the liquid control volume, and the ratio $\delta$ represents the portion of energy released during the interface condensation that is lost to the wall.

Another equation to be used in the lumped formulation is the volume balance equation, eqn.(3.11).

It is noted that $W_{STB}$, $W_{SRL}$, $W_{CI}$, $W_{EI}$, $W_{CD}$, $W_{BR}$, $Q_{WS}$, $Q_{WL}$, $Q_{TR}$, $Q_{PWR}$, $\beta$ and $\delta$ are calculated using analytical or empirical constitutive equations. Exceptions are when the drift-flux formulation is used, in this case, $W_{BR}$ and $W_{CD}$ are calculated using eqn.(3.9) and eqn.(3.10) respectively.

## 3.3 Drift-Flux Formulation

The basic concept of the drift-flux model is that the dynamics of the two-phase mixture as a whole can be expressed by a mixture momentum equation, and the relative motion between phases by a kinematic constitutive equation, namely the calculation of the drift-velocity $v_{dj}$:

$$v_{dj} = (1 - \langle\alpha_d\rangle)(\langle\langle v_d \rangle\rangle - \langle\langle v_c \rangle\rangle) \tag{3.18}$$

The time averaged three-dimensional form of the drift-flux model's equations have been derived by Ishii [1] Using area averaging, the equations can be converted into the following one-dimensional forms, and used as the governing equations in the drift-flux component of the IDRIFF code :

Mixture continuity equation :

$$\frac{\partial\langle\rho_m\rangle}{\partial t} + \frac{\partial}{\partial z}(\langle\rho_m\rangle\,\bar{v}_m) = 0\,; \tag{3.19}$$

Continuity equation for the dispersed phase :

$$\frac{\partial\langle\alpha_d\rangle\rho_d}{\partial t} + \frac{\partial}{\partial z}(\langle\alpha_d\rangle\rho_d\,\bar{v}_m) = \langle\Gamma_d\rangle - \frac{\partial}{\partial z}\left(\frac{\langle\alpha_d\rangle\rho_d\rho_c}{\langle\rho_m\rangle}\,v_{dj}\right); \tag{3.20}$$

Mixture momentum equation :

$$\frac{\partial\langle\rho_m\rangle\bar{v}_m}{\partial t} + \frac{\partial}{\partial z}(\langle\rho_m\rangle\,\bar{v}_m^2) = \frac{\partial}{\partial z}\langle P_m\rangle + \frac{\partial}{\partial z}\langle\tau_{zz} + \tau_{zz}^T\rangle$$

$$- \langle\rho_m\rangle g_z - \frac{f_m}{2D}\langle\rho_m\rangle\,\bar{v}_m|\bar{v}_m| - \frac{\partial}{\partial z}\left[\frac{\langle\alpha_d\rangle\rho_d\rho_c}{(1-\langle\alpha_d\rangle)\langle\rho_m\rangle}\,v_{dj}^2\right]$$

$$- \frac{\partial}{\partial z}\sum_k \mathrm{COV}(\alpha_k\rho_k v_k v_k) + \langle M_m\rangle; \tag{3.21}$$

Mixture enthalpy energy equation :

$$\frac{\partial\langle\rho_m\rangle\bar{h}_m}{\partial t} + \frac{\partial}{\partial z}(\langle\rho_m\rangle\,\bar{h}_m\,\bar{v}_m) = -\frac{\partial}{\partial z}\langle q + q^T\rangle + \frac{q_w^{\cdot}\,\xi_h}{A}$$

$$- \frac{\partial}{\partial z}\left[\frac{\langle\alpha_d\rangle\rho_d\rho_c}{\langle\rho_m\rangle}\,\Delta h_{dc}\,v_{dj}\right] - \frac{\partial}{\partial z}\sum_k \mathrm{COV}(\alpha_k\rho_k h_k v_k) +$$

$$+ \frac{\partial}{\partial z}\langle P_m\rangle + \left[\bar{v}_m + \frac{\langle\alpha_d\rangle\rho_d\rho_c}{\langle\rho_m\rangle}\,v_{dj}\right]\frac{\partial\langle\rho_m\rangle}{\partial z} + \langle\Phi_m^\mu\rangle + \langle\Phi_m^\sigma\rangle + \langle\Phi_m^i\rangle. \tag{3.22}$$

In the above, $\Gamma_d$ is the interfacial mass transfer term for the dispersed phase due to phase change, $\tau_{zz}$ and $\tau_{zz}^T$ are the normal components of the viscous stress and turbulent stress respectively, $(f_m/2D)\langle\rho_m\rangle\bar{v}_m|\bar{v}_m|$ is the two-phase frictional pressure drop term, q is the conduction heat flux, while $q^T$ is the turbulent diffusion flux of energy, $q_w''$ is the wall heat flux and $\xi_h$ is the heated perimeter; $\Delta h_{dc}$ is the difference between the specific enthalpy of the two phases. The covariance terms represent the difference between the average of a product and the product of the average of two variables such that

$$\mathrm{COV}(\alpha_k\rho_k\psi_k v_k) = \langle\alpha_k\rho_k\psi_k(v_k - \langle\langle v_k\rangle\rangle)\rangle$$

$M_m$ is the interfacial momentum source, $\phi_m^\mu$ is the viscous dissipation term; $\phi_m^\sigma$ is the work due to surface tension force and $\phi_m^i$ is the interfacial mechanical energy transfer

## 4.0 Major Numerical Algorithms

### 4.1 Equation of State

The equation of State is needed to close the calculations of the governing equations. In the lumped formulation, there are five equations to be solved, namely eqn.(3.11), eqn.(3.12), eqn.(3.13), eqn.(3.14) and eqn.(3.15). However, there are seven unknowns : $M_S$, $M_L$, $H_S$, $H_L$, $V_S$ (or $V_L$), $P_S$ and $P_L$. Two equations of state are needed, one for the liquid control volume and the other for the steam control volume. Similarly, in the drift-flux formulation, there are six unknowns namely $<\rho_m>$, $<\alpha_d>$, $\rho_d$, $\bar{v}_m$, $\bar{h}_m$ and $<P_m>$ to be solved at each grid-point from the four governing equations (eqn.(3.19) to (3.22)). An equation of state calculating the averaged mixture pressure is certainly needed. Furthermore, another equation of state calculating the dispersed phase pressure $P_d$ is added. Although with the addition of the final equation, which brings another unknown $P_d$, the total number of equations is still one less than the number of unknowns, the solving of the governing equation becomes less difficult. This will be discussed in the Section 4.3

A non-iterative equation of state in time derivative form developed recently [2] is implemented in IDRIFF. In the lumped formulation, since in each of the control volume there exists two phases at saturation under a uniform pressure, the form of equation of state suggested in [2] is adopted directly :

Steam control volume:

$$\frac{\partial P_{SA}}{\partial t} = G_1(P_{SA}, x_s)\frac{\partial h_S}{\partial t} + G_2(P_{SA}, x_S)\frac{\partial \rho_S}{\partial t} \tag{4.1a}$$

Liquid control volume :

$$\frac{\partial P_{LA}}{\partial t} = G_1(P_{LA}, x_L)\frac{\partial h_L}{\partial t} + G_2(P_{LA}, x_L)\frac{\partial \rho_L}{\partial t} . \tag{4.1b}$$

Here,

$$G_1(P,x) = \frac{v_g - v_f}{\left[x\frac{dh_g}{dp} + (1-x)\frac{dh_f}{dp}\right](v_g - v_f) - \left[x\frac{dv_g}{dp} + (1-x)\frac{dv_f}{dp}\right](h_g - h_f)} \tag{4.2}$$

and

$$G_2(P,x) = \frac{(h_g - h_f)/\rho^2}{\left[x\frac{dh_g}{dp} + (1-x)\frac{dh_f}{dp}\right](v_g - v_f) - \left[x\frac{dv_g}{dp} + (1-x)\frac{dv_f}{dp}\right](h_g - h_f)} \tag{4.3}$$

The values of saturated specific volumes, $v_g$ and $v_f$, saturated specific enthalpies $h_g$ and $h_f$ and their derivatives with respect to pressure are functions of pressure and are calculated using correlations presented in [3].

The local mixture pressure $<P_m>$ at each grid-point in the drift-flux calculation is also assumed to be the local saturation pressure at the grid-point. Hence the equation of state for $<P_m>$ is similar to that of eqn.(4.1):

$$\frac{\partial <P_m>}{\partial t} = G_1(P_m, x_m)\frac{\partial \bar{h}_m}{\partial t} + G_2(P_m, x_m)\frac{\partial <\rho_m>}{\partial t}, \tag{4.4}$$

except that $G_L$ and $G_2$ are calculated for each grid-point and are functions of local mixture presure $P_m$ and local mixture quality $x_m$.

On the other hand, the equation of state for the dispersed phase needs a little modification from that developed in [2]. From eqn.(3.1) and eqn.(3.2), it can be shown that

$$P_d = <P_m> + (1 - <\alpha_d>)\Delta P. \tag{4.5}$$

Taking the time derivative of eqn.(4.5) and assuming the dispersed particles' size does not vary significantly with pressure (hence the time rate of change of the local interphase pressure difference is negligibly small), the following can be written:

$$\frac{\partial P_d}{\partial t} = G_1(P_m, x_m)\frac{\partial \bar{h}_m}{\partial t} + G_2(P_m, x_m)\frac{\partial <\rho_m>}{\partial t} - \Delta P\frac{\partial <\alpha_d>}{\partial t} \tag{4.6}$$

where eqn(4.4) has been substituted.

## 4.2 Calculation of Quality and Swelling or Shrinking of the Control Volume

The rate of change of mass and energy in a control volume are trackable and hence can be calculated explicitly. However, the calculation of the rate of change of the volume is not straight forward. The uncertainty in the calculation of the volume imposes certain difficulties in the predictions of pressure and the quality in the control volume after a thermodynamic process. An algorithm involving certain iteration process is used in the IDRIFF. The development of the algorithm is based on the following consideration. It is recalled that pressure is calculated using the following general form of equation of state:

$$\frac{\partial P}{\partial t} = G_1\frac{\partial h}{\partial t} + G_2\frac{\partial \rho}{\partial t}. \tag{4.7}$$

The quality can be calculated from either of

$$x^{(1)} = \frac{(h - h_f)}{(h_g - h_f)} \tag{4.8}$$

$$x^{(2)} = \frac{(1/\rho - v_f)}{(v_g - v_f)}. \tag{4.9}$$

It was found that there is a small discrepency between $x^{(1)}$ and $x^{(2)}$. The discrepency is more significant after a rapid process. Physically, $x^{(1)}$ is more reliable since the averaged specific enthalpy h is calculated as the ratio of the total enthalpy and the total mass in the control volume, both are calculated explicitly. However, values of $h_f$ and $h_g$ are determined from the predicted value of pressure, the correctness of which has been influenced by the value of the averaged density $\rho$. The density $\rho$ is calculated as the ratio of the total mass and the total volume of the control volume and hence carries an error if the volume is not calculated correctly. On the other hand, both the error associated with $\rho$ and the errors in calculating $v_f$ and $v_g$, which are functions of pressure, contribute to the error in the value of $x^{(2)}$.

A series of numerical experiments were carried out by simulating control volumes (some dominated by the liquid phase (low quality) and some dominated by the gas phase (high quality)) undergoing a step change in the total mass and in the total enthalpy. At any time during the transient, the value of pressure is calculated as an instantaneous function of the current values of h and $\rho$:

$$P = P(h,\rho) \tag{4.10}$$

After a long transient, the final steady state value of the quality is very close to the one predicted based on averaged specific enthalpy (eqn.(4.8)). However, it is interesting to note that in the liquid phase dominated control volumes, the value of P throughout the transient and in the final steady state do not significantly depart from the one predicted by eqn.(4.7). On the other hand, in the gas phase dominated control volumes, there is a difference between the prediction based on eqn.(4.10) and that based on eqn.(4.8) whenever there is a disagreement between $x^{(1)}$ and $x^{(2)}$. This can be explained by the fact that the function $G_2$, which actually is the partial derivative of presure with respect to the density, $\partial P/\partial \rho$, does not significantly depend on pressure at low quality and changes rapidy with pressure at high quality condition.

It was mentioned in Section 3.1 that the liquid phase dominated control volume is assumed to be more incompressible than the steam control volume, which is dominated by the gas phase. This implies that in the pressurizer system, which has a fixed total volume, the swelling or shrinking of the liquid control volume is not significantly influenced by the swelling or shrinking of the steam control volume. On the other hand, the change in the volume of the steam control volume is almost entirely dictated by the change in the volume of the liquid control volume. This assumption, as well as the observation from the numerical experiments above, lead to the development of the algorithm for solving the lumped governing equations, which is listed below.

Liquid control volume:

1. The changes in $M_L$ and $H_L$ are calculated;
2. The change in $V_L$ is estimated by taking into account:
   (a) the additions and the substractions of volumes associated with the incoming and outgoing of flows into the liquid control volume;
   (b) the vertical motion of the liquid surface due to pressure difference in the steam control volume and in the liquid control volume. The latter is calculated by a simplified momentum balance equation;
3. New values of the $h_L$ and $\rho_L$ are calculated by taking the ratio of $H_L$ and $M_L$, and the ratio of $M_L$ and $V_L$ respectively;
4. New value of $P_{LA}$ is calculated using eqn.(4.1b);
5. New value of $x_L$ is calculated using the formula as in eqn.(4.8);

6. Through eqn.(4.9), the value of $\rho_L$ is corrected based on the value of $x_L$ calculated in the Step 5;

7. New value of $V_L$ is then calculated based on the corrected value of $\rho_L$ and the value of $M_L$ — this is the process of determining the swelling or shrinking of the liquid control volume;

Steam control volume:

8. New value of $V_S$ is calculated by substracting $V_L$ from the pressurizer total volume;

9. The changes in $M_S$ and $H_S$ are calculated;

10. New values of $h_S$ and $\rho_S$ are calculated by taking the ratio of $H_S$ and $M_S$ and the ratio of $M_S$ and $V_S$ respectively;

11. New value of $P_{SA}$ is estimated using eqn.(4.1a)

12. New value of $x_S$ is calculated using the formula as in eqn.(4.8);

13. Using eqn.(4.9), a value of $\rho_S$ is calculated based on values of $x_S$ calculated in the Step 12 and of $P_{SA}$ calculated in the Step 11;

14. A comparison is made between the value of $\rho_S$ calculated in the Step 13 and the actual one calculated in the Step 10;

15. If the discrepency between the two $\rho_S$'s is larger than a criterion, the value of $P_{SA}$ is corrected by using eqn.(4.1a) as if the averaged density has changed from $\rho_S$ calculated in the Step 13 to that calculated in the Step 10, then go back to the Step 12; Otherwise the calculation is completed.

## 4.3 Simultaneous Solutions of the Drift-flux Equations

The drift-flux governing equations (eqn.(3.19) to (3.22)) are basically solved as follows:

1. The main-parameter is identified from each of the four governing equations;

2. Using a modified Crank-Nicolson semi-implicit finite difference scheme, each of the governing equations is written as a tridiagonal matrix system, with the main-parameters of all grid-points as the unknown vector. The matrix system is solved and the values of the main-parameter at each grid-points are determined;

3. At each grid point, from the values of the four main parameters, as well as using eqn (4.4) and eqn.(4.6), values of all drift flux parameters are calculated algebraically.

To define the main-parameter and to demonstrate how they are identified, the mixture continuity equation (eqn.(3.19)) is used here as an example :

$$\frac{\partial\langle\rho_m\rangle}{\partial t} + \frac{\partial}{\partial z}(\langle\rho_m\rangle\, \bar{v}_m) = 0$$

The equation involves two parameters : $\langle\rho_m\rangle$ and $\bar{v}_m$. However, the equation is basically used to calculate value of $\langle\rho_m\rangle$. The parameter $\bar{v}_m$ appears in the second term to represent the convective effect. The value of $\bar{v}_m$ itself is basically calculated in the mixture momentum equation (eqn.(3.21)). Based on this consideration, $\langle\rho_m\rangle$ is identified as the main-parameter of the mixture continuity equation.

The modified Crank Nicolson scheme is basically a combination of an explicit finite difference scheme and the Crank Nicolson semi implicit scheme. In the explicit scheme, the space derivative is expressed by a central difference method. Hence the mixture continuity equation can be written as :

$$\frac{\langle\rho_m\rangle_{ij} - \langle\rho_m\rangle_{ij-1}}{\Delta t} + \frac{[(\langle\rho_m\rangle_{i+1j-1})(\bar{v}_{mi+1j-1}) - (\langle\rho_m\rangle_{i-1j-1})(\bar{v}_{mi-1j-1})]}{2\,\Delta z} = 0 \tag{4.11}$$

where $\Delta t$ and $\Delta z$ is respectively the increment in time and space. The subscript, i, represent the number of the grid-point in the axial direction and j represents the number of the time step.

On the other hand, using the Crank-Nicolson semi- implicit scheme, the same equation can be written as:

$$\frac{\langle\rho_m\rangle_{ij} - \langle\rho_m\rangle_{ij-1}}{\Delta t} + \frac{[(\langle\rho_m\rangle_{i+1j})(\bar{v}_{mi+1j}) - (\langle\rho_m\rangle_{i-1j})(\bar{v}_{mi-1j})]}{4\,\Delta z}$$

$$+ \frac{[(\langle\rho_m\rangle_{i+1j-1})(\bar{v}_{mi+1j-1}) - (\langle\rho_m\rangle_{i-1j-1})(\bar{v}_{mi-1j-1})]}{4\,\Delta z} = 0 \tag{4.12}$$

If j-1 represents the previous time step, there is only one unknown in eqn.(4.11), that is $\langle\rho_m\rangle_{ij}$. On the other hand, there are five unknowns in eqn.(4.12) : $\langle\rho_m\rangle_{i-1j}$, $\langle\rho_m\rangle_{ij}$, $\langle\rho_m\rangle_{i+1j}$, $\bar{v}_{mi+1j}$ and $\bar{v}_{mi-1j}$.

In the modified Crank-Nicolson scheme, only the main-parameters from eqn. (4.12) retains their semi-implicit expressions, the other are written in explicit forms. Another word, the values of $\bar{v}_m$ at current time step is approximated by their values from previous time step. Hence in eqn.(4.12), $\bar{v}_{mi+1j}$ is replaced by $\bar{v}_{mi+1j-1}$, and $\bar{v}_{mi-1j}$ is replaced by $\bar{v}_{mi-1j-1}$. This results in only the main-parameter: $\langle\rho_m\rangle_{i-1j}$, $\langle\rho_m\rangle_{ij}$ and $\langle\rho_m\rangle_{i+1j}$, are exclusively the unknowns.

This modification is justified when the model is used to simulate flow condition where the change in the mixture velocity is very small, such as in the case of pressurizer. After the modification, eqn.(4.12) can be written as:

$$-\left(\frac{\Delta t}{4\,\Delta z}\,\bar{v}_{mi-1j-1}\right)\langle\rho_m\rangle_{i-1j} + \langle\rho_m\rangle_{ij} + \left(\frac{\Delta t}{4\,\Delta z}\,\bar{v}_{mi+1j-1}\right)\langle\rho_m\rangle_{i+1j}$$

$$= \langle\rho_m\rangle_{ij-1} - \frac{\Delta t}{4\,\Delta z}[(\langle\rho_m\rangle_{i+1j-1})(\bar{v}_{mi+1j-1}) - (\langle\rho_m\rangle_{i-1j-1})(\bar{v}_{mi-1j-1})]. \qquad (4.13)$$

If N is the number of grid-points, the above equation can be written in a matrix form:

$$\bar{A}\;\bar{U}^I = \bar{B} \qquad (4.14)$$

where

$$\bar{A} = \begin{bmatrix} 1 & ar_1 & & & & & & \\ al_2 & 1 & ar_2 & & & 0 & & \\ & al_3 & 1 & ar_3 & & & & \\ & & - & - & - & & & \\ & & & & al_i & 1 & ar_i & \\ & 0 & & & & - & - & - \\ & & & & & & al_N & 1 \end{bmatrix},$$

$$\text{with} \quad ar_i = \frac{\Delta t}{4\,\Delta z}(\bar{v}_{mi+1j-1})$$

$$\text{and} \quad al_i = \frac{-\Delta t}{4\,\Delta z}(\bar{v}_{mi-1j-1}),$$

$$\bar{U}^I = \begin{bmatrix} u_1^I \\ u_2^I \\ . \\ . \\ u_i^I \\ . \\ u_N^I \end{bmatrix}, \qquad \bar{B} = \begin{bmatrix} b_1 \\ b_2 \\ . \\ . \\ b_i \\ . \\ b_N \end{bmatrix}$$

$u_i^I$ is the main parameter of the first governing equation, or the mixture continuity equation, at grid-point i:

$$u_i^I = \langle\rho_m\rangle_{ij}$$

and $b_i$ is the right hand side of the eqn.(4.13) :

$$b_i = <\rho_m>_{ij-1} - \frac{\Delta t}{4\,\Delta z} [<\rho_m>_{i+1j-1})(\bar{v}_{mi+1j-1}) - (<\rho_m>_{i-1j-1})(\bar{v}_{mi-1j-1})]$$

$b_1$ and $b_N$ are slightly modified to accomodate boundary condition.

This tridiagonal matrix system can be solved directly using a simple algorithm [4]. The result is the values of $u_i^I$, or $<\rho_m>_{ij}$.

The main parameters from the other governing equations can be identified in a similar way. They are listed on Table 4.1.

TABLE 4.1 List Of Main-parameters $u^k$, $k = I, IV$

| Governing Equations | Main Parameters |
|---|---|
| mixture continuity equation, Eqn. (3.19)<br>dispersed phase continuity equation, Eqn. (3.20)<br>mixture momentum equation, Eqn. (3.21)<br>mixture energy equation, Eqn. (3.22) | $u^I = \rho_m>$<br>$u^{II} = <\alpha_d> \rho_d$<br>$u^{III} = <\rho_m> v_m$<br>$u^{IV} = <\rho_m> h_m - P_m$ |

Using the modified Crank-Nicolson scheme described above, Eqn(3.20) to (3.22) are also arranged into tridiagonal matrix systems similar to Eqn(4.14), the unknown vevtor $U^k$ of each has components $u_i^k$ corresponding to the main-parameter listed on Table 4.1. The coefficient matrices $\bar{A}$ in all systems are identical. The systems are solved individually and $u_i^k$ are calculated.

Adopting the same (i,j) grid-point notation, the equations of state, Eqn.(4.4) and eqn.(4.6), can respectively be written as :

$$\begin{aligned} &<P_m>_{ij} - G_{1ij-1}\,\bar{h}_{mij} - G_{2ij-1}<\rho_m>_{ij} \\ &= <P_m>_{ij-1} - G_{1ij-1}\,\bar{h}_{mij-1} - G_{2ij-1}<\rho_m>_{ij-1} \end{aligned} \qquad (4.15)$$

and

$$\begin{aligned} &P_{d,ij} - G_{1ij-1}\,\bar{h}_{mij} - G_{2ij-1}<\rho_m>_{ij} + \Delta P_{ij-1}<\alpha_d>_{ij} \\ &= P_{d,ij} - G_{1ij-1}\,\bar{h}_{mij-1} - G_{2ij-1}<\rho_m>_{ij-1} + \Delta P_{ij-1}<\alpha_d>_{ij-1} \end{aligned} \qquad (4.16)$$

At each time step j, after solving the governing equations matrix systems and calculating the right hand sides of Eqn.(4.15) and Eqn.(4.16), the following can be obtained for each grid-point i :

$$<\rho_m>_{ij} = u_i^I; \qquad (4.17)$$

$$<\alpha_d>_{ij}\rho_{dij} = u_i^{II};\tag{4.18}$$

$$<\rho_m>_{ij}\bar{v}_{m,ij} = u_i^{III};\tag{4.19}$$

$$<\rho_m>_{ij}\bar{h}_{m,ij} - <P_m>_{ij} = u_i^{IV};\tag{4.20}$$

$$<P_m>_{ij} - G_{1ij-1}\bar{h}_{m,ij} - G_{2ij-1} - <\rho_m>_{ij} = u_i^{V};\tag{4.21}$$

and

$$P_{dij} - G_{1ij-1}\bar{h}_{mij} - G_{2ij-1}<\rho_m>_{ij} + \Delta P_{ij-1}<\alpha_d>_{ij} = u_i^{VI},\tag{4.22}$$

where $u_i^V$ denotes the right-hand-side of the Eqn.(4.15) and $u_i^{VI}$ denotes the right-hand-side of the Eqn.(4.16).

There are seven unknowns, namely $<\rho_m>$, $<\alpha_d>$, $\rho_d$, $\bar{v}_m$, $\bar{h}_m$, $<P_m>$ and $P_d$, to be solved from the above six equations. An iterative approach is used : First, the value of $<\alpha_d>$ is guessed, and the rest of the parameters are solved algebraically. Next, since the dispersed phase is assumed to be saturated corresponding to the local dispersed phase pressure $P_d$, a value of $\rho_d$ can be calculated as the dispersed phase saturation density at $P_d$. This is compared with the value of $\rho_d$ calculated from Eqn.(4.18). Any discrepency is used to guide the correction of $\alpha_d$, and another iteration is repeated. The effectiveness of the iteration procedure strongly depends on the first guess of $\alpha_d$. Two different approaches are used in this respect, depending on the control volume. In the steam control volume, since the dispersed phase (liquid-phase) density is almost constant with pressure, the previous value of $\rho_d$ at a grid-point i is used to obtain the first guessed value of $<\alpha_d>$ at i from the Eqn.(4.18). On the other hand, the first guesing of $<\alpha_d>$ in the liquid control volume is less straight-forward. Taking the time derivative of the Eqn.(3.3), the following is obtained :

$$\frac{\partial<\rho_m>}{\partial t} = <\alpha_d>\frac{\partial\rho_d}{\partial t} + \rho_d\frac{\partial<\alpha_d>}{\partial t} + (1 - <\alpha_d>)\frac{\partial\rho_c}{\partial t} - \rho_c\frac{\partial<\alpha_d>}{\partial t}$$

The third term on the right-hand-side can be dropped since changes on the continuum phase (liquid-phase) density is negligibly small. The following can therefore be written :

$$\begin{aligned}&<\rho_m>_{ij} - <\alpha_d>_{ij-1}\rho_{dij} - (\rho_{dij-1} - \rho_{cij-1})<\alpha_d>_{ij}\\&\quad = <\rho_m>_{ij-1} - <\alpha_d>_{ij-1}\rho_{dij-1} - (\rho_{dij-1} - \rho_{cij-1})<\alpha_d>_{ij-1}\end{aligned}\tag{4.23}$$

It is noted that there are three unknowns in the Eqn (4.23), namely $<\rho_m>_{ij}$, $\rho_{dij}$ and $<\alpha_d>_{ij}$. Hence the first guess of $<\alpha_d>$ can be obtained by solving Eqn.(4.17), Eqn.(4.18) and Eqn.(4.23) simultaneously

## 5.0 Application of the IDRIFF to the Pressurizer Analysis

Many pressurizer models are reported in the literature or currently used in the nuclear industry. Examples are those developed by Gorman [5], Nahavandi and Makkenchery [6] and Baggoura and Martin [7]. In most of them, the thermodynamic state(s) of the fluid in the pressurizer is pre-assumed. Mathematical models are then set-up based on this assumption. The models are usually adequate in predicting pressurizer behaviour qualitatively. However, inconsistent performance of the models for different power plant transients is generally reported [8]. It is believed that the major drawback of these pressurizer models is due to the lack of understanding on the detail physical phenomena in the pressurizer. The objective of the present pressurizer study is to provide such an understanding, especially that of local interphasial mass and energy exchanges. Two paths of study are undertaken in order to achieve this objective. First, empirical experiments on pressurizer behaviour are performed on a laboratory scale. The flow-regimes of the fluid in the pressurizer are visually observed. Measurements of pressure, temperature, void-fraction and flow in as many locations as technically possible are taken. Second, numerical simulations of the experiments are done using the IDRIFF code, providing further detail information about the pressurizer behaviour.

The experimental loop is basically as shown in the Figure 1. The pressurizer itself is a glass tank with an inside-diameter of 5.1 cm and a height of about 66 cm. Five immersion type electrical heaters are installed at the bottom of the tank. The heat generated from the heaters can be adjusted to a maximum of 1 Kw. A control valve and an adjustable relief valve on top of the tank are used to control a steam-bleed flow and hence to control the steam pressure in the pressurizer. Three pressure transducers, three thermocouples and three capacitance probes are installed on the pressurizer. The capacitance probes are used to measure relative void-fraction of the fluid. Experimental data are sent to a PDP-11 computer and are processed by a data acquisition system.

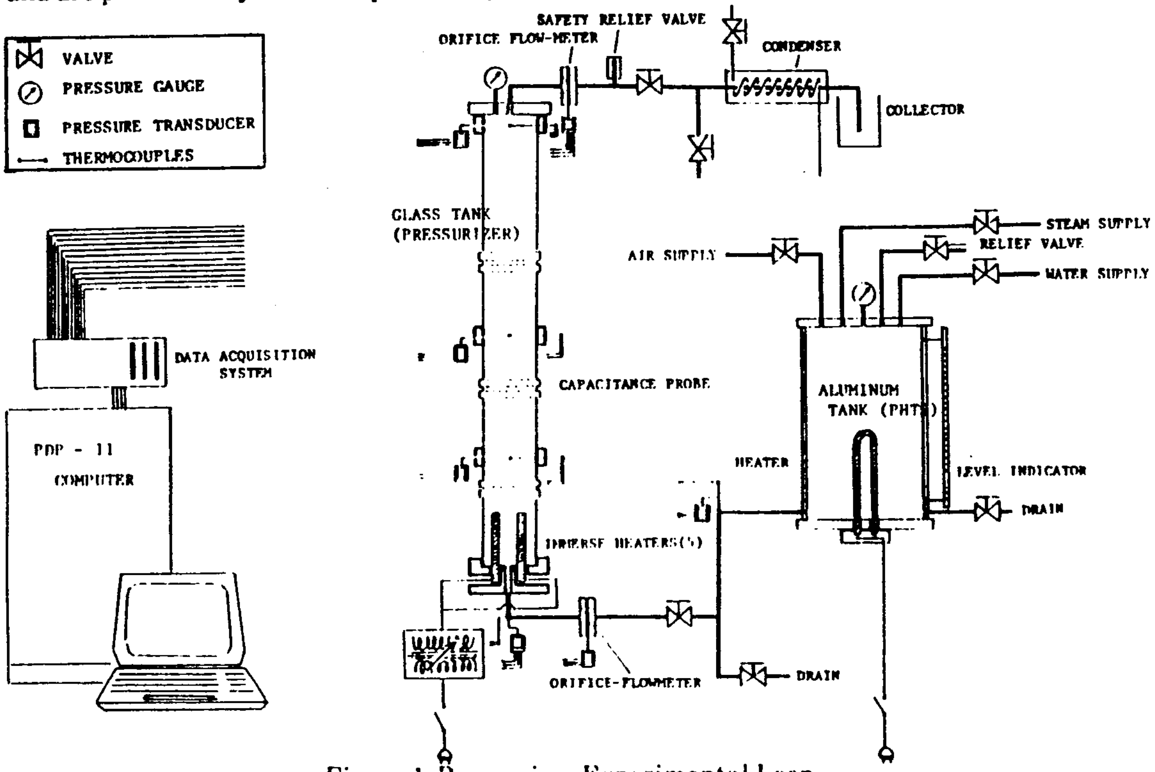


Figure 1 Pressurizer Experimental Loop

The first part of the experiment is concentrated on the quasi-steady state conditions of the pressurizer in isolation [9]. By controlling the heater power, liquid level and steam pressure to some constant values, the pressurizer fluid eventually reaches a quasi steady state, in which heat input from the heaters is approximately balanced by the sum of heat loss from pressurizer tank wall and energy carried out by the steam bleed flow. The flow regimes of the fluid under the quasi steady state are observed. The results are mapped in the Figure 2, where the experimental control parameters, heaters power, liquid level and steam pressure, are directly used as the coordinates of the map. When the analysis of the data is completed, a generalized flow-regime map with more appropriate parameters (dimensionless) as the coordinates will be presented.

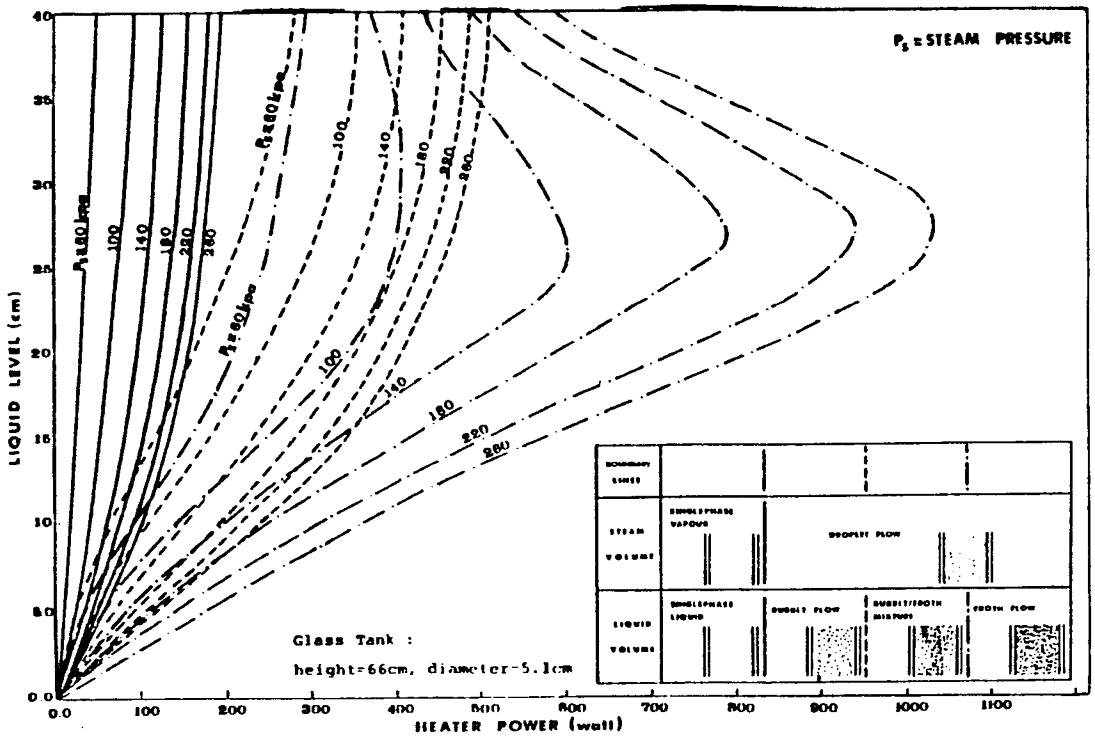
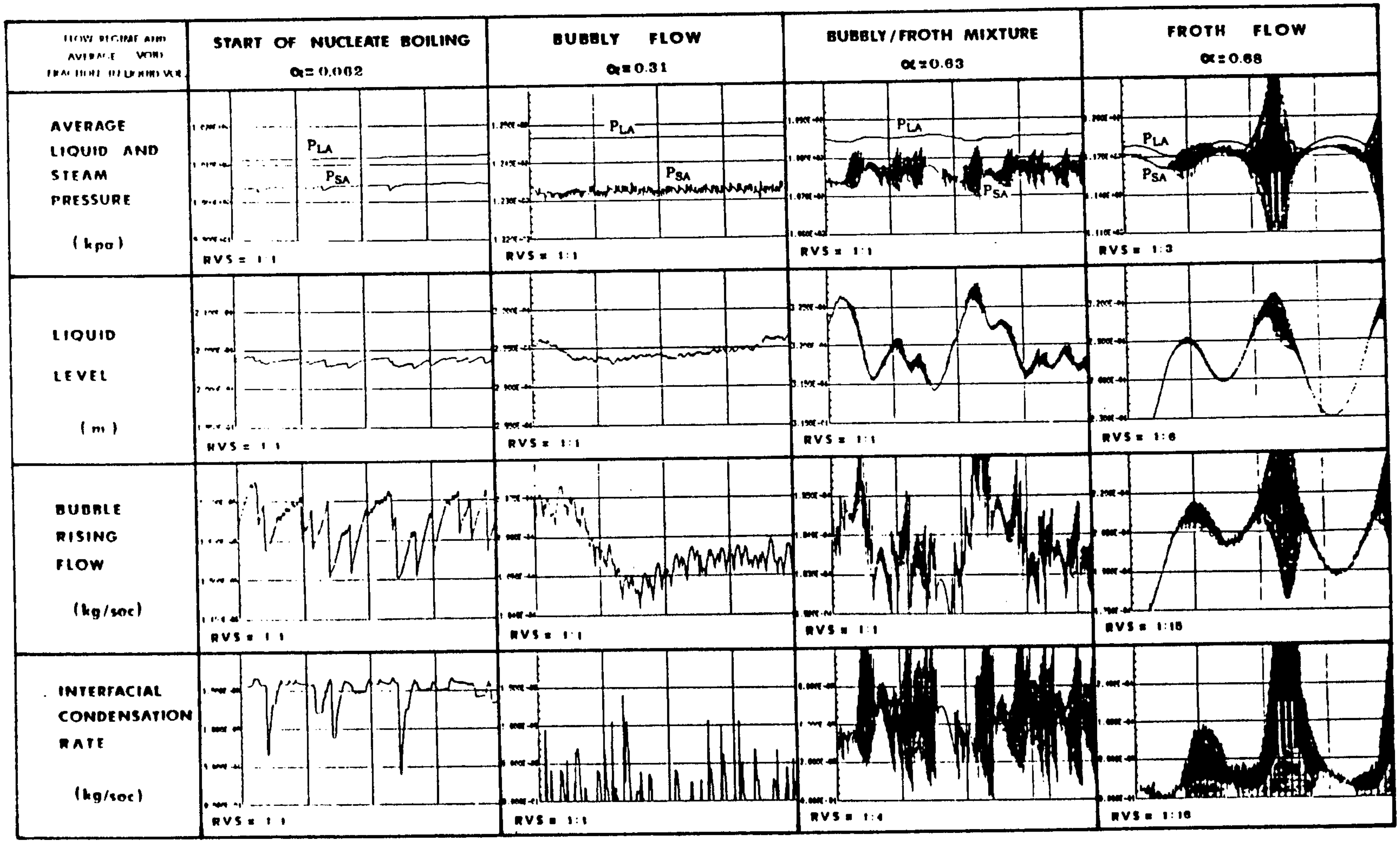


Figure 2 Map of Pressurizer Quasi-steady state Flow regimes

The quasi-steady state conditions are simulated by using the transient calculations of the IDRIFF code. The values of the steam pressure, liquid pressure (with correction due to any hydrostatic pressure difference), liquid temperature and liquid level are used as initial conditions for the simulation. Values of heater power and constant values of environment conditions are used as the boundary conditions. Each of the simulations is carried out until the corresponding quasi-steady state is reached, where the predicted values of all parameters such as pressures, temperatures and liquid level quasi-steadily match those measured in the experiment. The transient values of these parameters are recorded. At various times, detail profiles of the parameters as calculated by the drift-flux calculation are also recorded.

At present, some 78 quasi-steady state experimental conditions are being simulated and the results are being analyzed. Preliminary results are very encouraging. Distinct physical phenomena are predicted in various different flow-regimes. Examples of these are shown in the Figure 3. The behaviour of these parameters reveal much information regarding the local mechanical and thermal non-equilibrium in the fluid as well as the overall non-equilibrium between the liquid and the steam in the pressurizer.



Horizontal axis : TIME    scale : 2 sec.

RVS = Relative vertical axis scale

Figure 3 Typical Behaviour of Pressurizer Parameters at Various Flow Regimes.

## 6.0 Summary

The basic description of the IDRIFF code has been presented in this paper. The provision of both the lumped formulation and the drift-flux formulation in the code enables it to be used as a flexible and powerful tool in the analysis of a two-phase system such as a pressurizer. Preliminary qualitative results from the application of the IDRIFF code to the analysis of pressurizer has been briefly discussed. More complete and quantitative results will be reported in the near future.

## Acknowledgement

The authors wish to thank the Science and Engineering Research Board of McMaster University for its financial support of this work.

## References

1. M.Ishii, "Thermo-Fluid Dynamic Theory Of Two-phase Flow", chapter IX and X, Eyrolles, Paris(1975)
2. R.Sollychin, S.A.Adebiyi and W.J.Garland, "The Development Of A Non-Iterative Equation Of State For Two-phase Flow Systems", 11th CNS Simulation Symposium, Kingston, April 1985
3. J.Hoskins and W.J.Garland, "Approximation Functions For The Calculations Of Thermodynamic Properties And The Equation Of State For Light Water At Saturation", 2nd McMaster Univ. Nuclear Symposium, Oct.1985
4. D.Greenspan, "Discrete Numerical Methods In Physics And Engineering", chapter 1, Academic Press Inc.(1974)
5. D.J. Gorman, "Steam Surge Tank Transient During OUtsurg", ASME 09-WA/NE-14, 1969
6. A.N. Nahavandi, S. Makkenchery, "An Improved Pressurizer Model with Bubble Rise and Condensate Drop Dynamics", Nuc. Eng. & Design, 12, 1970.
7. B. Baggoura, W. Martin, "Transient Analysis of the Three Mile Island Unit 2 Pressurizer System", Nuclear Technology, vol. 62, 1983.
8. S.M. Sami, "A Dynamic Model for Predicting CANDU Pressurizers Performance", Nuclear Technology, vol. 72, 1986
9. R.Sollychin, W.J.Garland and J.S.Chang, "Study Of Pressurized Bubbly Column Using Drift-flux Model", Physics Society Of Japan Annual Meeting, Chiba, Oct.1985