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MULTIPLE TEMPORAL-MODE ANALYSIS FOR THREE-DIMENSIONAL REACTOR DYNAMICS

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Abstract

A multiple temporal-mode transformation combined with spatial differencing is developed and applied in a two-group, three-dimensional reactor dynamics analysis using the diffusion approximation. It is found that this solution formalism leads to an apparently efficient and effective method for the study of fast transients for time intervals before a global flux trend has been established. Calculational results for a simplified cylindrical reactor under conditions of a fast transient and some general properties of the formalism are discussed.

Introduction

The development of efficient and effective analytical and calculational models which describe a wide range of dynamic characteristics of a nuclear reactor in three dimensions has long represented a desirable goal. Experience with low-dimensional calculations involving suitably chosen analytical descriptions and numerical strategies suggests that this goal may be attainable.

The three-dimensional, multi-group, time-dependent analysis of a nuclear reactor generally provides a basis for considerable scope in exploring various strategies. Factors such as available computational resources, detail of system description, temporal domain of interest, and others combine to provide various though restricted options.

Here we describe a three-dimensional, two-group, time-dependent neutron diffusion analysis based on the use of a multiple temporal-mode transformation combined with finite differencing in space. As will become clear, the chosen analytical formalism seems particularly applicable for fast transients; in this context, this defines the dynamics of a nuclear reactor before a dominant global flux trend has been established. In the following we will describe the solution formalism and relate it to a dynamic discretization. Finally, some preliminary calculational results will be presented and some general features of the solution formalism will be identified.

Multiple-Mode Solution Representation

The three-dimensional, multi-group, time-dependent neutron diffusion description of a nuclear reactor with or without delayed neutron can be represented in symbolic form by

$$\frac{\partial}{\partial t}\,\underline{\psi}(\underset{\rightarrow}{r},t) = \underline{\underline{A}}\,\underline{\psi}(\underset{\rightarrow}{r},t)\ , \tag{1}$$

where $\underline{\underline{A}}$ is a systems matrix and $\underline{\psi}(\underset{\rightarrow}{r},t)$ is the vector containing the neutron group fluxes and the neutron precursor groups. We choose to base our analysis on a functional prescription used effectively by Hansen and associates (1-3) and restrict our solution to the form

$$\psi(t_{i+1}) \sim \psi'(t_i)\exp[\alpha\Delta t_i]\ , \tag{2}$$

for a time step $\Delta t_i = t_{i+1} - t_i$. Here, α is a suitable parameter valid during Δt.

To be specific, we consider the two-group, time-dependent equation in three-dimensions written in its common form

$$\frac{1}{v_g}\frac{\partial}{\partial t}\phi_g(\underset{\rightarrow}{r},t) = \underset{\sim}{\nabla}\cdot D_g(\underset{\rightarrow}{r},t)\underset{\sim}{\nabla}\phi_g(\underset{\rightarrow}{r},t) + \sum_{g'=1}^{2}\Sigma_{gg'}(\underset{\rightarrow}{r},t)\phi_{g'}(\underset{\rightarrow}{r},t) \ , \qquad (3)$$

for g = 1,2 and where the symbols possess the usual meaning. Note that herein we do not include delayed neutron precursors although the solution formalism to be described here can be readily extended to include this effect as well as more than two groups. We interject to note that the exclusion of delayed neutron precursors will clearly effect the characteristic temporal evolution of the neutron flux in a calculational context. The above equations, Eq. (3), are coupled implying that a solution which retains this coupling explicitly is desirable. Thus we choose to use an Ansatz given by

$$\phi_g(\underset{\rightarrow}{r},t) = \Phi_g(\underset{\rightarrow}{r},t)\sum_{\ell=1}^{2}\beta_{g\ell}(\underset{\rightarrow}{r})\exp[\alpha_\ell(\underset{\rightarrow}{r})t] \ , \qquad (4)$$

for $t \in t_j$. Here $\phi_g(\underset{\rightarrow}{r},t)$ may be called a transformed flux while $\beta_{g\ell}(\underset{\rightarrow}{r})$ and $\alpha_\ell(\underset{\rightarrow}{r})$ may be termed moments and frequencies respectively. We emphasize two important properties of this Ansatz. First, the flux-coupling property specifies here that the g'th flux, $\phi_g(\underset{\rightarrow}{r},t)$, is given in terms of the moments and frequencies associated with all other neutron group terms. Second, the proposed solution, Eq. (4), reduces to the standard point-kinetics solution if no spatial dependence is assumed to exist.

To place the multi-mode Ansatz, Eq. (4), in historical perspective we point out that it does possess some relation to the Ansatz previously used by Reed et al (1) and Wight et al (2)

$$\phi_g(\underset{\rightarrow}{r},t) = \psi_g(\underset{\rightarrow}{r},t)\exp[\alpha_g(\underset{\rightarrow}{r})t] \ , \qquad (5)$$

and that used by Ferguson and Hansen (3)

$$\phi_g(\underset{\rightarrow}{r},t) = \psi_g(\underset{\rightarrow}{r},t)\exp[\alpha(\underset{\rightarrow}{r})t] \ . \qquad (6)$$

The distinction in these three solution models rests primarily in the extent to which the time variations of the various neutron groups are explicitly listed as contributors in effecting a change in any given group flux.

Analytical Development

In the analysis to be pursued here, we restrict ourselves to an arbitrary time step, $\Delta t_j = t_{j+1} - t_j$, for which the system parameters, $D_g(\underset{\rightarrow}{r})$ and $\Sigma_{gg'}(\underset{\rightarrow}{r})$, are assumed to be given as a function of position only. Thus, our describing equation for the g'th neutron energy group is now given by

$$\frac{1}{v_g}\frac{\partial}{\partial t}\phi_g(\underset{\rightarrow}{r},t) = \underset{\sim}{\nabla}\cdot D_g(\underset{\rightarrow}{r})\underset{\sim}{\nabla}\phi_g(\underset{\rightarrow}{r},t) + \sum_{g'=1}^{2}\Sigma_{gg'}(\underset{\rightarrow}{r})\phi_{g'}(\underset{\rightarrow}{r},t) \ , \qquad (7)$$

where again g = 1,2 and t ∈ Δt. Substituting Eq. (4) into Eq. (7) yields, after some algebraic rearrangement, the describing equations in terms of $\Phi_g(\underset{\rightarrow}{r},t)$, $\beta_{g\ell}(\underset{\rightarrow}{r})$ and $\alpha_\ell(\underset{\rightarrow}{r})$:

$$\sum_{\ell=1}^{2} \exp[\alpha_\ell(\underset{\rightarrow}{r})t]\{\frac{\beta_{g\ell}(\underset{\rightarrow}{r})}{v_g} \frac{\partial}{\partial t} \Phi_g(\underset{\rightarrow}{r},t)\} = -\sum_{\ell=1}^{2} \exp[\alpha_\ell(\underset{\rightarrow}{r})t]\{\frac{\beta_{g\ell}(\underset{\rightarrow}{r})\alpha_\ell(\underset{\rightarrow}{r})}{v_g} \Phi_g(\underset{\rightarrow}{r},t)\}$$

$$+ \sum_{\ell=1}^{2} \beta_{g\ell}(\underset{\rightarrow}{r})\exp[\alpha_\ell(\underset{\rightarrow}{r})t]\{\underset{\sim}{\nabla}.D_g(\underset{\rightarrow}{r})\underset{\sim}{\nabla}\Phi_g(\underset{\rightarrow}{r},t)\} + \underset{\sim}{\nabla}.D_g(\underset{\rightarrow}{r})\Phi_g(\underset{\rightarrow}{r},t)\underset{\sim}{\nabla} \sum_{\ell=1}^{2} \exp[\alpha_\ell(\underset{\rightarrow}{r})t]\beta_{g\ell}(\underset{\rightarrow}{r})$$

$$+ \sum_{\ell=1}^{2} \exp[\alpha_\ell(\underset{\rightarrow}{r})t]\{\sum_{g'=1}^{2} \Sigma_{gg'}(\underset{\rightarrow}{r})\beta_{g'\ell}(\underset{\rightarrow}{r})\Phi_{g'}(\underset{\rightarrow}{r},t)\} \quad . \tag{8}$$

Writing the transformed describing equations in this form identifies the common multiplier containing the ℓ'th order frequency, $\exp[\alpha_\ell(\underset{\rightarrow}{r})t]$, in each but the third term of the right hand side of this equation; the "del" operator prevents a convenient extraction of this exponential term. Although an argument could be attempted that the contribution of this term might be small and hence could be neglected, we propose to set this term equal to zero on the grounds that the resultant equation will be used only to evaluate the moments, $\beta_{g\ell}(\underset{\rightarrow}{r})$, and the frequencies, $\alpha_\ell(\underset{\rightarrow}{r})$, at $t_i = 0$. This latter imposition possesses the feature that it renders the terms upon which $\underset{\sim}{\nabla}$ operates a constant and hence defines the entire 3'rd term equal to zero. We will comment on this point again in a subsequent section. Thus, viewing the leading term, $\exp[\alpha_\ell(\underset{\rightarrow}{r})t]$, as a common coefficient requires that the ℓ'th term in each summation be equated as follows:

$$\frac{\beta_{g\ell}(\underset{\rightarrow}{r})}{v_g} \frac{\partial}{\partial t} \Phi_g(\underset{\rightarrow}{r},t) = -\frac{\beta_{g\ell}(\underset{\rightarrow}{r})}{v_g} \alpha_\ell(\underset{\rightarrow}{r})\Phi_g(\underset{\rightarrow}{r},t)$$

$$+ \underset{\sim}{\nabla}.D_g(\underset{\rightarrow}{r})\underset{\sim}{\nabla}\Phi_g(\underset{\rightarrow}{r},t) + \sum_{g'=1}^{2} \Sigma_{gg'}(\underset{\rightarrow}{r})\beta_{g'\ell}(\underset{\rightarrow}{r})\Phi_{g'}(\underset{\rightarrow}{r},t) \quad , \tag{9}$$

for g = 1,2 and t ∈ Δt_i. This equation will now be examined to permit the evaluation of $\Phi_g(\underset{\rightarrow}{r},t)$, $\beta_{g\ell}(\underset{\rightarrow}{r})$, and $\alpha_\ell(\underset{\rightarrow}{r})$ at all discrete values of $\underset{\rightarrow}{r}$ and for ℓ = 1,2.

Finite Analysis

We consider arbitrary time steps $\Delta t_i = t_{i+1} - t_i$ and, for convenience, choose to use $t_i = 0$ for all iterative time intervals. Either from the time when the simulation is initiated or from the beginning of an arbitrary iteration in time, we set $t = t_i = 0$ and use Eq. (4) to write

$$\phi_g(\underset{\rightarrow}{r},0) = \Phi_g(\underset{\rightarrow}{r},0) \sum_{\ell=1}^{2} \beta_{g\ell}(\underset{\rightarrow}{r}) \ . \tag{10}$$

We choose to use a normalization on the moments $\beta_{g\ell}(\underset{\rightarrow}{r})$ defined by

$$\sum_{\ell=1}^{2} \beta_{g\ell}(\underset{\rightarrow}{r}) = 1 \ , \tag{11}$$

and thus have

$$\Phi_g(\underset{\rightarrow}{r},0) = \phi_g(\underset{\rightarrow}{r},0) \ , \tag{12}$$

at the beginning of each time step.

To find the moments and frequencies, we use the value, $\Phi_g(\underset{\rightarrow}{r},0)$, associated with the beginning of each interval and assume that the temporal evolution of the neutron flux during the interval Δt_i is predominantly exponential. That is, in Eq. (9) we impose the following condition on the transformed flux $\Phi_g(\underset{\rightarrow}{r},t)$

$$\frac{\partial}{\partial t} \Phi_g(\underset{\rightarrow}{r},t)\Big|_{t \in \Delta t_i} = 0 \ , \tag{13}$$

and obtain an equation in terms of $\beta_{g\ell}(\underset{\rightarrow}{r})$, $\alpha_\ell(\underset{\rightarrow}{r})$ and the initial conditions, $\Phi_g(\underset{\rightarrow}{r},0)$:

$$-\beta_{g\ell}(\underset{\rightarrow}{r})\alpha_\ell(\underset{\rightarrow}{r}) + \beta_{g\ell}(\underset{\rightarrow}{r}) v_g \frac{\underset{\sim}{\nabla}\cdot D_g(\underset{\rightarrow}{r}) \underset{\sim}{\nabla} \Phi_g(\underset{\rightarrow}{r},0)}{\Phi_g(\underset{\rightarrow}{r},0)}$$

$$+ \sum_{g'=1}^{2} \Sigma_{gg'}(\underset{\rightarrow}{r}) \beta_{g'\ell}(\underset{\rightarrow}{r}) v_g \frac{\Phi_{g'}(\underset{\rightarrow}{r},0)}{\Phi_g(\underset{\rightarrow}{r},0)} = 0 \tag{14}$$

This equation can be written as an eigenvalue equation for the frequencies $\alpha_1(r)$ and $\alpha_2(r)$ corresponding to the values of $D_g(\underset{\rightarrow}{r})$ and $\beta_{g\ell}(\underset{\rightarrow}{r})$ at $\underset{\rightarrow}{r}$ and $\Phi_g(\underset{\rightarrow}{r},0)$ about $\underset{\rightarrow}{r}$:

$$\begin{bmatrix} \Sigma_{11}(\underset{\rightarrow}{r})v_1 + \dfrac{v_1 \underset{\sim}{\nabla}\cdot D_1(\underset{\rightarrow}{r}) \underset{\sim}{\nabla}\Phi_1(\underset{\rightarrow}{r},0)}{\Phi_1(\underset{\rightarrow}{r},0)} & \Sigma_{12}(\underset{\rightarrow}{r})v_1 \dfrac{\Phi_2(\underset{\rightarrow}{r},0)}{\Phi_1(\underset{\rightarrow}{r},0)} \\ \Sigma_{21}(\underset{\rightarrow}{r})v_2 \dfrac{\Phi_1(\underset{\rightarrow}{r},0)}{\Phi_2(\underset{\rightarrow}{r},0)} & \Sigma_{22}(\underset{\rightarrow}{r})v_2 + \dfrac{v_2 \underset{\sim}{\nabla}\cdot D_2(\underset{\rightarrow}{r}) \underset{\sim}{\nabla}\Phi_2(\underset{\rightarrow}{r},0)}{\Phi_2(\underset{\rightarrow}{r},0)} \end{bmatrix} \begin{bmatrix} \beta_{1\ell}(\underset{\rightarrow}{r}) \\ \beta_{2\ell}(\underset{\rightarrow}{r}) \end{bmatrix} = \alpha_\ell(\underset{\rightarrow}{r}) \begin{bmatrix} \beta_{1\ell}(\underset{\rightarrow}{r}) \\ \beta_{2\ell}(\underset{\rightarrow}{r}) \end{bmatrix} \tag{15}$$

or, symbolically,

$$\underset{\approx}{C}\underset{\sim}{\beta}_\ell = \alpha_\ell \underset{\sim}{\beta}_\ell \ . \tag{16}$$

Since all elements of $\underline{\underline{C}}$ are assumed known at the coordinate of interest, $\vec{r}$, the eigenvalues α_ℓ can be found by the solution to

$$|\underline{\underline{C}} - \alpha_\ell \underline{\underline{I}}| = 0 \ , \tag{17}$$

In this two-group representation, $\alpha_1(r)$ and $\alpha_2(r)$, are the two solutions of a quadratic equation.

The components of the eigenvector β_ℓ, Eq. (17), are given by any non-zero column of the adjoint of $[\underline{\underline{C}}-\alpha_\ell \underline{\underline{I}}]$, $\text{ADJ}[\underline{\underline{C}}-\alpha_\ell \underline{\underline{I}}]$, to within an arbitrary constant. Supposing that these vector components are identified by $\beta'_{g\ell}(\vec{r})$. Then, according to the normalization condition, Eq. (12), we find a constant h_g, g = 1,2, such that

$$h_g[\beta'_{g1}(\vec{r}) + \beta'_{g2}(\vec{r})] = 1 \ . \tag{18}$$

Hence, the moments $\beta_{g\ell}(\vec{r})$, g = 1,2, ℓ = 1,2, are determined. With the functions $\Phi_g(\vec{r},0)$, $\alpha_\ell(\vec{r})$ and $\beta_{g\ell}(\vec{r})$ all known at $t_i = 0$, we insert these values into Eq. (8) and, by finite numerical methods solve for $\Phi_g(\vec{r},t_{i+1})$. With the use of Eq. (4), this resultant expression yields the flux $\phi_g(\vec{r},t_{i+1})$ at the end of the time step.

Numerical Modeling and Calculational Results

For the purpose of an initial exploratory three-dimensional examination of our formalism, it is advantageous to adopt a simple and direct numerical model and to concentrate on relatively large scale effects. Our simplified three-dimensional reactor is therefore chosen to be a homogeneous, bare cylinder penetrated by a control rod perpendicular to the axial direction.

We choose to restrict ourselves to a coarse spatial mesh describing the whole reactor with 1320 mesh points. Errors may arise from three sources: 1) numerical round-off, 2) finite difference expressions, and 3) boundary condition approximations.

Round-off errors will not be a severe problem in our calculation both because of the precision of the long (60 bit) capacity of the CDC-6400 computer used in our calculation and because our equations involve only tridiagonal (three-stripe) matrices. Computer memory size limits the number of values of the flux which can be conveniently stored; in particular, we cannot keep the value of the flux at many past times and thus are forced to compute the flux at time t based on values of the flux and system parameters at time t-Δt and accept the attendant truncation error in the time step. If the boundaries of the reactor do not lie on our chosen grid points then we must take an approximation to the actual boundary values; we can avoid this error, however, by describing our reactor in cylindrical co-ordinates (r,θ,z).

We have chosen to describe the spatial domain by two diametrically opposite 90° wedges of the core. Use of the physical bi-lateral symmetry of our reactor model allows us to construct all values not explicitly lying within our grid thus reducing our computer storage requirements. The control rod has been made to appear more regular by weighting the parameters accordingly within the control rod region. A grid with 12 points in the r-direction, 10 points in the θ-direction and 11 points in the z-direction yielding 1320 grid points has been chosen.

Starting with some initial array of values for the flux $\Phi_g(r_i) = \Phi_g(r_i,\theta_i,z_i)$, g = 1,2, we solve Eq. (17) - which here represents a quadratic equation - for the spatial frequencies $\alpha_\ell(\underset{\rightarrow}{r}_i)$ and for the spatial moments $\beta_{g\ell}(\underset{\rightarrow}{r}_i)$ according to Eq. (11) and Eq. (18). The diffusion term in Eq. (15) is represented at the point $\underset{\rightarrow}{r}_i = (r_i,\theta_i,z_i)$ in finite different form. Equation (8) may subsequently be written as a tri-diagonal system which is implicit in one of the three spatial directions and explicit in the remaining two. In solving this system we have chosen to use the alternating two-direction implicit method (2).

The data used in the sample calculation here are listed in Table I and correspond approximately to those appropriate for a heavy water reactor. Figure 1 shows the fast and thermal flux rise associated with two sudden withdrawals of the control rod described by the following. At steady state we consider the axial flux traverse adjacent to the cylindrical axis with the control rod inserted 55% of the full cylindrical diameter. At t = 0 the control rod is suddenly withdrawn so that only 27.5% of the diametrical distance of the core contains the control rod. After a time interval of 320 μs the control rod is withdrawn entirely. The flux rise is followed for another 320 μs up to t = 640 μs, Fig. 1.

We note that during this very fast transient, the induced reactivity disturbance effects both the fast and thermal flux only in the region close to the control rod position. We also observe a contrast in the flux perturbation and response between the fast and thermal flux.

The computer time requirements for this three-dimension, two-group, 1320 mesh-point calculation was found to be 0.84 s/step on the CDC-6400 computer used here. This is almost identical to the computer time requirements reported by Reed (5) for an efficient two-dimensional calculation using a faster computer (CDC-6600).

Discussion of Solution Formalism

The describing equation, Eq. (1), used here can be easily extended to G energy groups and include I delayed neutron precursor groups. For this case we write the vector $\underset{\rightarrow}{\psi}(\underset{\rightarrow}{r},t)$ and the matrix $\underset{=}{A}$ in Eq. (1) as

$$\underline{\psi}(\underset{\rightarrow}{r},t) = [\psi_1, \psi_2, \ldots, \psi_G, C_1, \ldots, C_I]^T, \tag{19}$$

and

$$\underline{\underline{A}} = \left[\begin{array}{ccc:ccc} D_1 + T_{11} & & T_{1G} & F_{11} & \cdots & F_{1I} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ T_{G1} & & D_G + T_{GG} & F_{G1} & \cdots & F_{GI} \\ \hdashline P_{11} & \cdots & P_{1G} & -\Lambda_1 & & 0 \\ \vdots & \ddots & \vdots & & \ddots & \\ P_{I1} & \cdots & P_{IG} & 0 & & \Lambda_I \end{array}\right] \tag{20}$$

Here, the notation conforms to that used by Ferguson and Hansen (3). The matrix $\underline{\underline{A}}$ is real, irreducible, square and "essentially positive"; these properties are independent of the number of energy groups or delayed neutron precursor groups.

The transformation used in our work herein may be represented by

$$\underline{\psi} = \underline{\underline{\Omega}}\underline{\Phi} \quad , \tag{21}$$

where the matrix operator $\underline{\underline{\Omega}}$ is defined by

$$\underline{\underline{\Omega}} = \left[\begin{array}{ccc} \sum_{\ell=1}^{G} \beta_{1\ell}(\underset{\rightarrow}{r})\exp[\alpha_\ell(\underset{\rightarrow}{r})t] & & 0 \\ & \ddots & \\ 0 & & \sum_{\ell=1}^{G} \beta_{G\ell}(\underset{\rightarrow}{r})\exp[\alpha_\ell(\underset{\rightarrow}{r})t] \end{array}\right] \tag{22}$$

which, upon substitution in Eq. (1) yields,

$$\frac{\partial}{\partial t}\underline{\Phi} = \underline{\underline{\Omega}}^{-1}(\underline{\underline{A}} - \dot{\underline{\underline{\Omega}}}\underline{\underline{\Omega}}^{-1})\underline{\underline{\Omega}}\underline{\Phi} \; . \tag{23}$$

The moments $\beta_{g\ell}(\underset{\rightarrow}{r})$, and the frequencies $\alpha_\ell(\underset{\rightarrow}{r})$, are found from the condition

$$\frac{\partial}{\partial t}\underline{\Phi} = \underline{\underline{\Omega}}^{-1}(\underline{\underline{A}} - \dot{\underline{\underline{\Omega}}}\underline{\underline{\Omega}}^{-1})\underline{\underline{\Omega}}\underline{\Phi} = 0 \; . \tag{24}$$

This relationship suggests the following condition

$$(\underline{\underline{A}}'\underline{\underline{\Omega}} - \dot{\underline{\underline{\Omega}}})\underline{\Phi} = 0 \ , \tag{25}$$

for the determination of the moments and frequencies; here $\underline{\underline{A}}'$ is simply $\underline{\underline{A}}$ in which $\nabla.D(\vec{r})\nabla$ is evaluated using the flux at t = 0. This eigenvalue equation, Eq. (25) is, of course, the same equation as Eq. (17) obtained by a different algebraic method.

It is important to note that the difference matrix $(\underline{\underline{A}} - \underline{\underline{A}}')$, appearing by subtraction involving Eq. (1)

$$\frac{\partial}{\partial t}\underline{\Phi} = \underline{\underline{\Omega}}^{-1}(\underline{\underline{A}} - \underline{\underline{A}}')\underline{\underline{\Omega}}\underline{\Phi} \ , \tag{26}$$

is composed only of terms containing the Laplacian ∇^2 along the main diagonal (4). Thus, the equations are separable in the neutron group flux and can be solved by sequential elimination. This is a most useful consequence since it eliminates the need for group scanning (3).

Some additional points of observation seem pertinent to this solution formalism. As indicated, the moments and frequencies, $\beta_{g\ell}(\vec{r})$ and $\alpha_\ell(\vec{r})$, are calculated from $\underline{\underline{A}}'$, that is, the matrix $\underline{\underline{A}}$ appropriate to the beginning of the time step. An improved value for $\underline{\underline{A}}'$ might be obtained using a predictor-corrector scheme although it appears that the benefits from such a scheme would be highly dependent upon the reactor system of interest.

Further, we approximated the fourth term in Eq. (8)

$$T_4 = \underline{\nabla}.D_g(\vec{r})\Phi_g(\vec{r},t)\underline{\nabla}\sum_{\ell=1}^{G}\exp[\alpha_\ell(\vec{r})t]\beta_{g\ell}(\vec{r}) \ , \tag{27}$$

at t = 0 while permitting the rest of the equation to be a function of time. This equation could be expanded to yield

$$T_4 = \sum_{\ell=1}^{G}\exp[\alpha_\ell(\vec{r})t]\beta_{g\ell}(\vec{r})\Gamma(\vec{r},t) \ , \tag{28}$$

where $\Gamma(\vec{r},t)$ is obtained by performing the spatial operations suggested in Eq. (27). Equation (9) could be expanded to include this additional term and therefore provide for an improved estimate for the moments and frequencies in Eq. (17).

Although the calculational experience with this new formalism is limited, it does appear to possess considerable merit particularly in the descriptions of fast reactor transients.

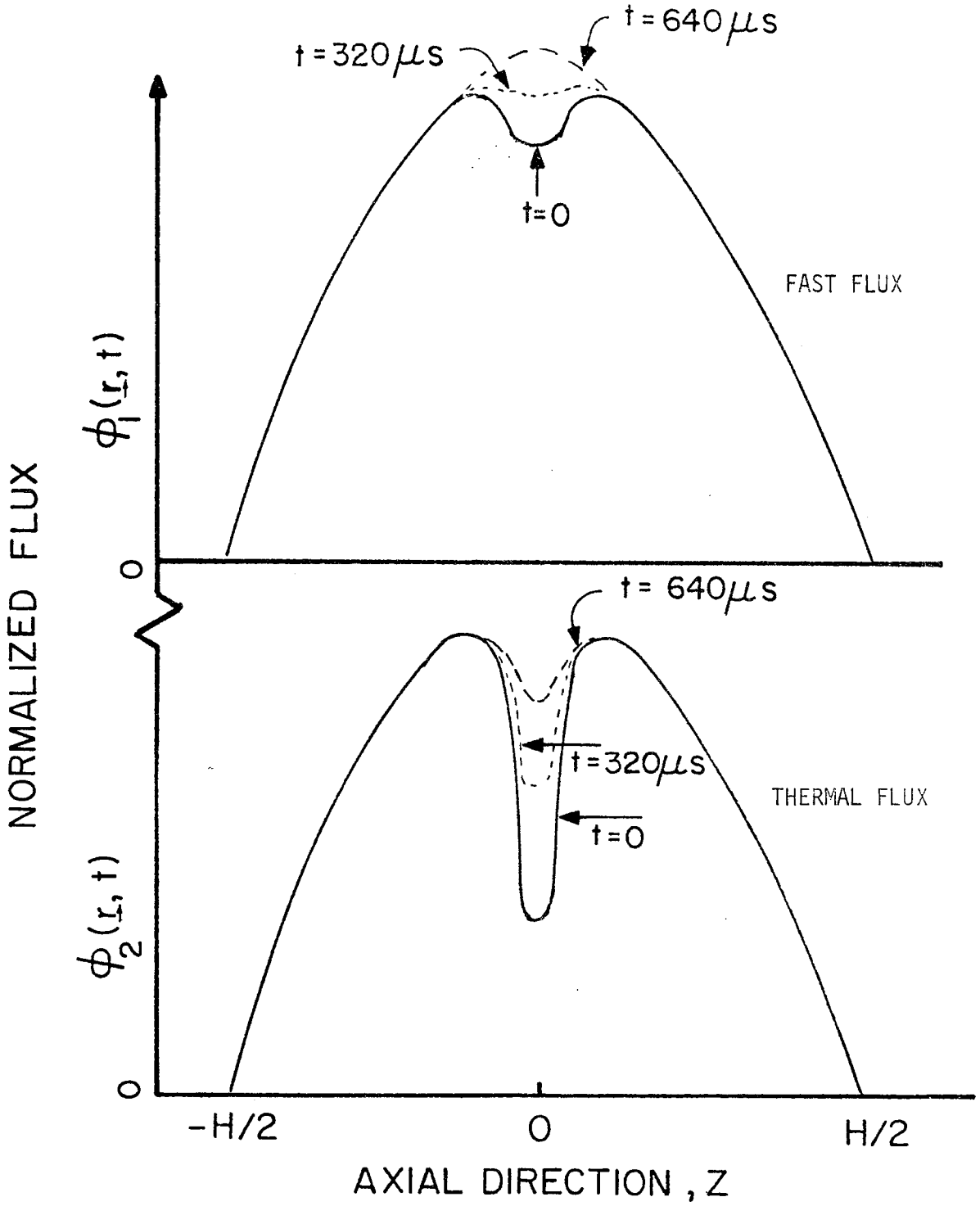

Fig. 1: Fast and thermal neutron flux rise. At t = 0 the control rod is suddenly raised from 55% of full diametrical insertion in core to 27.5%; at t = 320 μs the rod is withdran entirely. During the 640 μs time interval, the flux perturbations are found to be contained close to the region of the radial direction of control rod location.

Parameter	Reactor Core	Control Rod
Σ_{11} (cm^{-1})	-0.97×10^{-3}	-0.97×10^{-3}
Σ_{22} (cm^{-1})	-0.41×10^{-2}	-0.10×10^{0}
Σ_{21} (cm^{-1})	0.86×10^{-2}	0.86×10^{-2}
Σ_{12} (cm^{-1})	0.48×10^{-2}	0.48×10^{-2}
D_1 (cm)	1.40	1.30
D_2 (cm)	0.92	0.86
$v_1 = 2.0 \times 10^6$ cm/s ; $v_2 = 2.0 \times 10^5$ cm/s		
R = 3.50 m	H = 5.94 m	

Table I: Data used in sample calculation.

References

1. W.M. Reed and K.F. Hansen, "Alternating Direction Methods for Reactor Kinetics Equations", Nucl. Sci. Eng., 41, 431 (1970).

2. A.L. Wight, K.F. Hansen and D.R. Ferguson, "Application of Alternating-Direction Implicit Methods to the Space-Dependent Kinetics Equations", Nucl. Sci. Eng., 44, 239 (1971).

3. D.R. Ferguson and K.F. Hansen, "Solution of the Space-Dependent Reactor Kinetics Equations in Three Dimensions", Nucl. Sci. Eng., 51, 189 (1973).

4. W.J. Garland, A.A. Harms and J. Vlachopoulos, "Space-Time Nuclear Reactor Analysis by Temporal Transformation", Nucl. Sci. Eng., 55, 119 (1974).

5. W.H. Reed, "Finite Difference Techniques for the Solution of the Reactor Kinetics Equations", Sc.D. Thesis, Massachusetts Institute of Technology, Mass. (1969).

DISCUSSION

A. Siebertz

Your method could be called a quasi-static method with specialized exponential form amplitude function.

Could you comment on the relative merits of both methods (accuracy, computing time ...)?

A.A. Harms

First, for very rapid changes with highly localized effects, this method seems more accurate and at least as fast. Second, the one additional important merit point is associated with the group-coupling solution representation which describes an arbitrary group flux in terms of all other groups; thus, increases in one group and concurrent decreases in another group can be both described.

A.F. Henry

The improvement caused by these methods when applied to alternating directions schemes is much greater than when they are applied to fully implicit schemes.

D.A. Meneley

1) The quasi-static method is best suited to fast reactor problems.

2) Hansen's and Harm's methods have pointwise or/and groupwise exponential transformations with extrapolated coef-

ficients. The quasi-static method has a single amplitude function obtained by solution of the point kinetic equation. Quasi-static methods are presently limited to linear variation of shape functions with time; on the other hand, the Hansen-Harm's methods are limited in maximum time step length because of the extrapolation, constant properties over the time step, and by alternating-direction solution method. The question can only be answered by running a series of benchmark problems with all methods.

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PROCEEDINGS OF THE JOINT NEACRP/CSNI SPECIALISTS' MEETING ON NEW DEVELOPMENTS IN THREE-DIMENSIONAL NEUTRON KINETICS AND REVIEW OF KINETICS BENCHMARK CALCULATIONS

Garching, 22nd - 24th January 1975

The Specialists' Meeting was organized by the OECD Nuclear Energy Agency Committee on Reactor Physics (NEACRP), the Committee on the Safety of Nuclear Installations (CSNI) and the Laboratorium für Reaktorregelung und Anlagensicherung der Technischen Universität München (LRA)

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ABSTRACT

The papers presented at the meeting discuss problems related to three-dimensional kinetics calculations of nuclear reactors with the inclusion of feedback effects.

Solutions of benchmark problems posed by NEACRP/CSNI and submitted by various contributors are compared and discussed.

INTRODUCTION

The main topics of the papers are the following:

1. Recent developments of computational methods for the analysis of 3-d neutron kinetics:

   - Numerical methods general.
   - Coarse mesh methods.
   - Analysis and evaluation of 3-d neutron kinetcs calculations.

2. Comparison and discussion of benchmark problems posed by NEACRP/CSNI:

   - Four 1-d benchmark problems for a gas-cooled thermal reactor (9 submitted solutions)
   - 2-d benchmark problem for a LWR (5 submitted solutions)
   - 3-d benchmark problem for a LWR (2 submitted solutions)
   - Four 2-d benchmark problems for a fast reactor (4 submitted solutions)

The meeting was attended by 41 officially accepted participants from 15 countries and international organisations. 14 papers have been presented from 8 different countries and organisations.

W. Werner

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Session I

NUMERICAL METHODS GENERAL

Chairman: H. Küsters, Germany

Session II

COARSE MESH METHODS

Chairman: F.J. Fayers, United Kingdom

Session III

EVALUATION OF MULTIDIMENSIONAL CALCULATIONS

Chairman: W. Werner, Germany

Session IV/1

REVIEW OF KINETICS BENCHMARK CALCULATIONS

Chairman: A.F. Henry, USA

Session IV/2

REVIEW OF KINETICS BENCHMARK CALCULATIONS

Chairman: D.A. Meneley, Canada