# TRANSACTIONS

#### OF THE AMERICAN NUCLEAR SOCIETY 1974 ANNUAL MEETING

Bellevue Stratford Hotel

June 23-27, 1974

Philadelphia, Pennsylvania

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Since  $\partial \theta / \partial t = K \psi$  for the adiabatic reactor, the flux can be obtained from Eq. (13) as

$$\psi^{0}(\mathbf{x}, t) = \frac{\mathbf{p}^{2}/\mathbf{K} \cos\left(\frac{\pi}{2} \mathbf{x}\right)}{\frac{8\alpha}{3\pi} - \mathbf{p}\mathbf{g}\left[\exp(\mathbf{p}t) + \left(\frac{4\alpha}{3\pi}\right)^{2} \frac{\exp(-\mathbf{p}t)}{\mathbf{p}\mathbf{g}}\right]}, \quad (15)$$

yielding

$$\lim_{t \to \infty} \psi^{0}(x, t) = 0$$
 (16)

regardless of the change in  $\lambda.$  This intrinsic shutdown mechanism of the adiabatic reactor also has been shown by Kastenberg.  $^6\,$ 

Much information essential for the safety analysis of fast reactors can be obtained from the solutions obtained above. In spite of the intrinsic shutdown mechanism of the adiabatic reactor, its flux reaches a maximum several decades higher than that of the Newtonian reactor before dropping off. For the Newtonian feedback, the gas-cooled reactor reaches an asymptotic flux several magnitudes below that of the sodium-cooled reactor. Other information such as the rate of flux increase and the time at which the maximum flux occurs can be readily obtained and compared for various reactors.

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# 8. A Summation-Exponent Analysis for Space-Dependent Reactor Transients, W. J. Garland, J. Vlachopoulos, A. A. Harms (Mc-Master Univ)

Hansen et al.<sup>1,2</sup> have recently demonstrated the effectiveness of two methods of extracting the dominant exponential time dependence in space-dependent reactor transient calculations. In their first analysis,<sup>1</sup> the g'thgroup neutron flux in the space-time multigroup diffusion formulation is written as

$$\phi_{\mathbf{g}}(\mathbf{r}, t) = \psi_{\mathbf{g}}(\mathbf{r}, t) \exp(\alpha_{\mathbf{g}}t)$$
; (1)

while in a subsequent analysis, the special case of Eq. (1),

$$\phi_g(\mathbf{r}, t) = \psi_g(\mathbf{r} \cdot t) \exp(\alpha t)$$
 , (2)

is employed.

We have studied these two approaches and find that a summation-exponent-type representation can have a significant effect in the reduction of computer calculation time and/or increasing numerical accuracy. We consider the g'th-group neutron flux in the form

$$\begin{split} \phi_{\mathbf{g}}(\mathbf{r}, \, \mathbf{t}) &= \sum_{l=1}^{G} \, \psi_{\mathbf{g}, \mathbf{l}}(\mathbf{r} \, \cdot \, \mathbf{t}) \, \exp(\alpha_{\mathbf{l}} \mathbf{t}) \\ &= \psi_{\mathbf{g}}(\mathbf{r}, \, \mathbf{t}) \, \sum_{l=1}^{G} \, f_{\mathbf{g}, \mathbf{l}} \, \exp(\alpha_{\mathbf{l}} \mathbf{t}) \quad , \end{split} \tag{3}$$

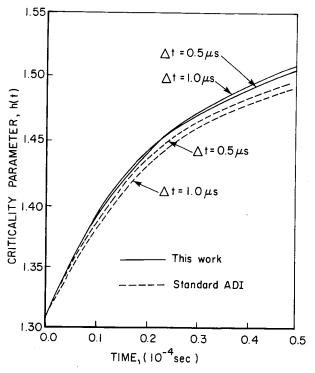


Fig. 1. Criticality parameter, h(t), as a function of time and time steps for the case of constant power productivity.

where we have specified G energy groups and, hence, G expansion coefficients and exponents. A fundamental and most significant consequence of this representation is that—depending on the relative "stiffness" of the diffusion equations—a most effective means of incorporating rapid changes in the system description is possible.

As an illustrative example, we consider a CANDU-BLW lattice cell modeled by a two-group neutron diffusion representation.<sup>3</sup> The initial flux distributions were assumed to be flat. The two-group diffusion equations were solved by the alternating direction implicit (ADI) method<sup>4</sup> using the transformation of Eq. (3) and, for comparison purposes, without the transformation. As a case of interest, we simulated the control situation in which the criticality parameter,

$$h(t) = \epsilon \, \eta \, f(t) \qquad , \tag{4}$$

was adjusted to maintain a constant power productivity, defined by

$$\int_{\text{fuel }} h(t) \, \phi_{\text{th}}(\mathbf{r}, t) \, d\mathbf{r} = \text{constant} \quad . \tag{5}$$

The result of this analysis is shown in Fig. 1. Of interest in this particular case is the observation that the summation-exponent representation, Eq. (3), provides consistently better numerical accuracy than the standard ADI method. The effective computation time saving was on the order of a factor of 3 to 4.

In addition to several calculational tests designed to study the effectiveness of the summation-exponent solution approach, we have also investigated the property of stability and convergence; these properties are met, in general, to the same extent as those applicable to the use of Eqs. (1) and (2). Finally, we have examined some criteria that can be used to provide qualitative conclusions of the effectiveness of this solution formalism based on the neutronic properties of the system.

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## 9. A Higher Order Relationship Between Static Power Tilts and Eigenvalue Separation, W. D. Beckner, R. A. Rydin (U of Va)

An important parameter in the analysis of spatial kinetics problems is the difference between the two largest eigenvalues of the static reactor balance equation. A method of accurately measuring eigenvalue separation would be useful to confirm calculational models used extensively in reactor design. Wade and Rydin have proposed measuring eigenvalue separation through a static flux tilt technique. This method uses the fact that, to first order, eigenvalue separation is inversely proportional to the flux tilt per unit of perturbing reactivity.

A higher order perturbation theory relationship has been developed here. Previous work has shown that the perturbed flux can be expanded as a linear combination of the lambda modes, with the expansion coefficients given by

$$A_{j} = \frac{\lambda_{0}}{\lambda_{0} - \lambda_{j}} \frac{\langle \Psi_{j}^{*}, (\delta M - \delta L)\phi \rangle}{\langle \Psi_{j}^{*}, \frac{M_{0}}{\lambda_{j}} \Psi_{j} \rangle} , \quad (1)$$

where  $\delta M$  and  $\delta L$  are the perturbations introduced in the production and destruction operators. A first-order approximation to these coefficients was obtained by assuming that the perturbed flux,  $\phi$ , is proportional to the fundamental mode,  $\Psi_0$ . This derivation also requires that the net reactivity introduced by the perturbation be zero.<sup>2</sup>

In an actual reactor, a tilt-inducing perturbation results in a nonzero reactivity, and a compensating change in reactor properties must accompany this perturbation to achieve criticality; e.g., a uniform change in soluble poison. The total perturbation can then be represented as the sum of the initial perturbation and the compensating perturbation. Now the coefficients become

$$A_{j} = \frac{\lambda_{0}}{\lambda_{0} - \lambda_{j}} \frac{C_{j} \rho}{\left(1 + \rho \frac{\lambda_{j} \lambda_{0}}{\lambda_{0} - \lambda_{j}}\right)} , \qquad (2)$$

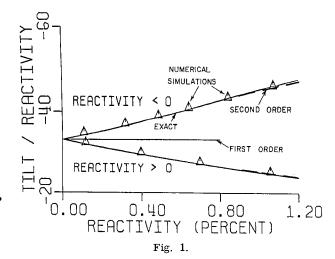
where

$$C_{j} = \frac{\rho_{j}}{\rho} = \frac{\langle \Psi_{j}^{*}, (\delta M - \delta L)\phi \rangle}{\langle \Psi_{0}^{*}, (\delta M - \delta L)\phi \rangle} , \qquad (3)$$

and where  $\rho$  is the change in  $k_{\hbox{\it eff}}$  induced by the initial perturbation. For small reactivities compared to the eigenvalue separation this form reduces to the original representation.

The previous work also assumed that the perturbed flux was similar to the original flux when calculating the correction factor,  $C_j$ . For small perturbations,  $C_j$  is approximately equal to  $C_j^0$ , where

$$C_{j}^{K} = \frac{1}{\lambda_{k}} \frac{\langle \Psi_{j}^{*}, (\delta M - \delta L) \Psi_{k} \rangle}{\langle \Psi_{o}^{*}, (\delta M - \delta L) \Psi_{o} \rangle} \quad . \tag{4}$$



For perturbations that significantly affect the flux shape, this approximation is not valid. By expanding the perturbed flux in Eq. (3) using the expansion coefficients given in Eq. (2), the following representation for the correction factors is found:

$$C_{j} = \frac{C_{j}^{0} + \rho \sum_{k=1}^{\infty} \frac{\lambda_{0} \lambda_{k}}{\lambda_{0} - \lambda_{k}} \frac{C_{k} C_{j}^{k}}{\left(1 + \rho \frac{\lambda_{0} \lambda_{k}}{\lambda_{0} - \lambda_{k}}\right)}}{1 + \rho \sum_{k=1}^{\infty} \frac{\lambda_{0} \lambda_{k}}{\lambda_{0} - \lambda_{k}} \frac{C_{k} C_{0}^{k}}{\left(1 + \rho \frac{\lambda_{0} \lambda_{k}}{\lambda_{0} - \lambda_{k}}\right)}$$
(5)

A second-order approximation is obtained by substituting the first-order approximation that  $C_j = C_j^0$  into Eq. (5). Higher order approximations are found by continuing this process. It has been found that this iteration converges very rapidly for realistic perturbations.

By substituting these two changes into the original theory, the tilt induced by a perturbation for a given eigenvalue separation can be calculated. The figure shows the predicted power tilt calculated by the original first-order theory compared to second-order theory and an "exact" prediction found by taking the iteration on the  $C_j$ 's to as many terms as necessary to get the desired accuracy. Data from computer simulations of power tilts on one-dimensional models are included in the figure. Agreement with the higher order theory is within 2% for several types of perturbations over a wide range of perturbation magnitudes.

In a practical situation, the seemingly complicated theory presented here simplifies greatly. Since the correction of the  $C_j$  terms is small, Eq. (5) can be truncated with little loss of accuracy. This new theory also suggests practical experimental procedures that would provide for more accurate interpretation of data by making the effects of higher harmonic modes less important than in the first-order theory; this analysis has been successfully applied to the SHA data.<sup>3</sup>

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