Overview Concept Map

Key concepts:

- heat is generated in the fuel at a rate proportional to reactor power

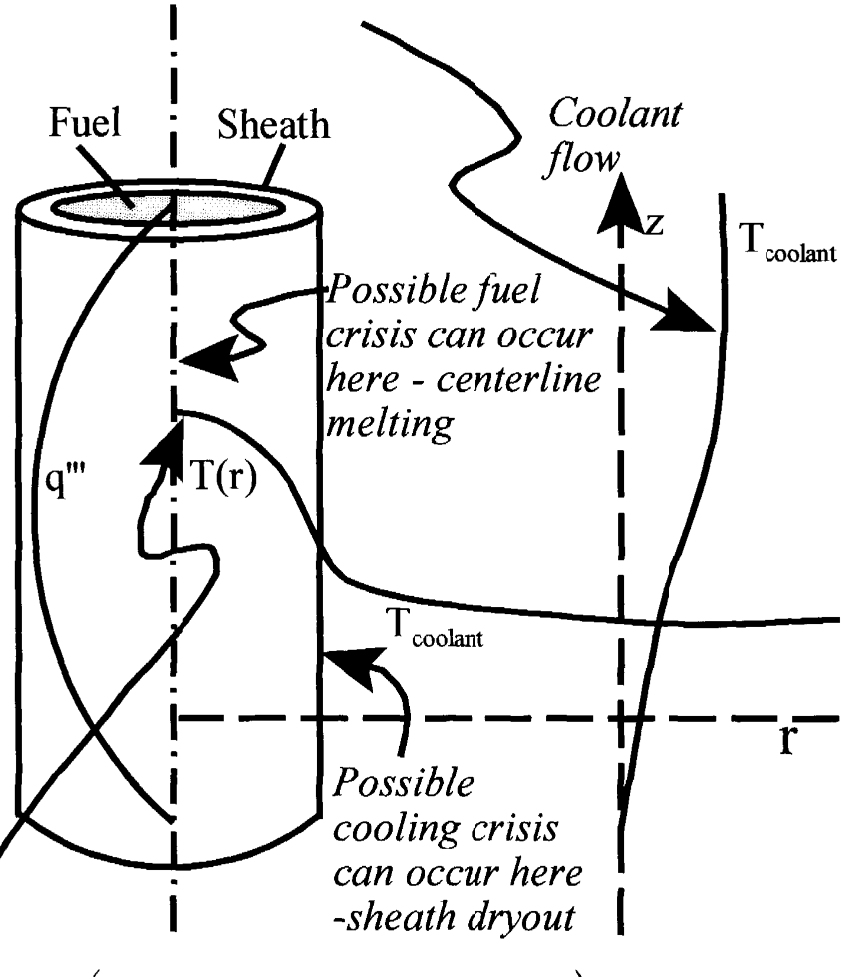
- heat is conducted to the coolant as the coolant flows along the fuel

- in the steady state, the fuel temperature is just sufficiently greater than the coolant temperature to transfer the heat generated - ie, fuel temperature "floats" on coolant temperature

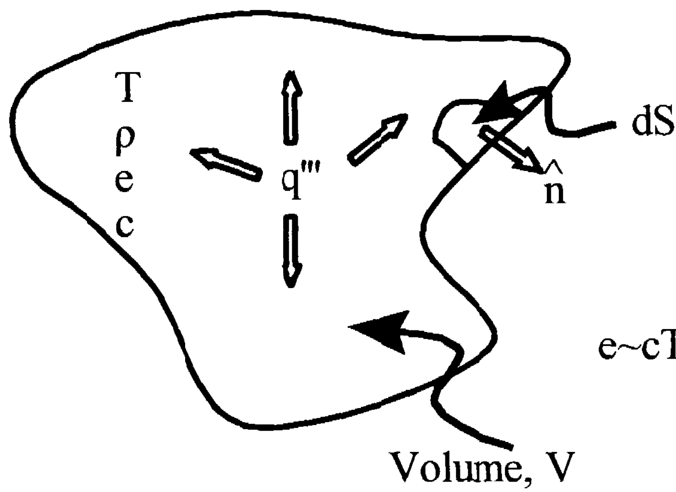
- too much power will cause the fuel temperature to be too high and a heat transfer crisis will occur

- heat transfer is a key limiting factor to power output

$$T_{FL}(z) - T_{INLET} = \frac{q_0 H}{\pi c W}\left(\sin\left(\frac{\pi z}{H}\right) + 1\right)$$



$$T_0 - T_{FL} = \frac{q'}{2\pi}\left(\frac{1}{2k_f} + \frac{1}{h_G r_F} + \frac{t_G + t_C}{k_C(r_F + t_G)} + \frac{1}{h_S(r_F + t_G + t_C)}\right)$$



Energy Balance:

Rate of change = Source - Sink

$$\iiint_V \frac{\partial(\rho e)}{\partial t} = \iiint_V q'''(\underline{r},t)\,dV - \iint_S \underline{q}''(\underline{r},t)\cdot\hat{n}\,ds$$

e~cT+const.

$$\iint_S \underline{J}\cdot\hat{n}\,ds = \iiint_V \nabla\cdot\underline{J}\,dV$$

$$\frac{\partial(\rho c T)}{\partial t} = q'''(\underline{r},t) - \nabla\cdot\underline{q}''(\underline{r},t)$$

$$\underline{q}''(\underline{r},t) = -k\nabla T(\underline{r},t)$$

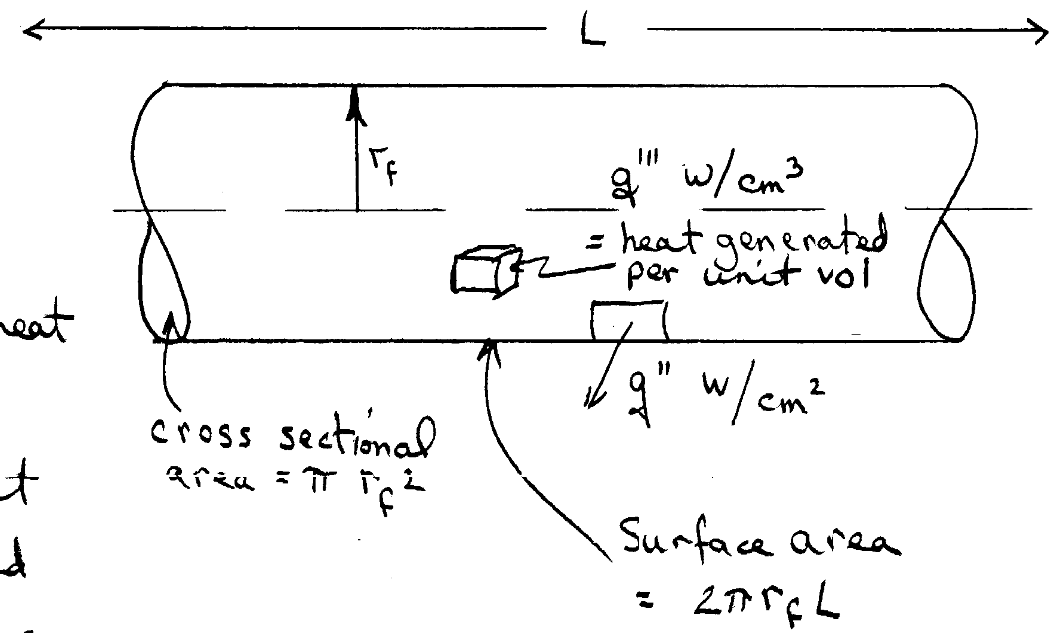
Fourier's Law

$$\frac{\partial(\rho c T)}{\partial t} = q'''(\underline{r},t) + \nabla\cdot k\nabla T(\underline{r},t)$$

Procedure:
- apply Gauss' Law to convert surface integral to volume integral
- replace e with cT
- drop volume integral
- apply Fourier's Law

Physical Setup:

- Long thin pencils
  ∴ neglect axial conduction
- heat generation in fuel meat
- radial conduction
- in steady state, <u>all</u> heat generated is conducted through the pin surface.

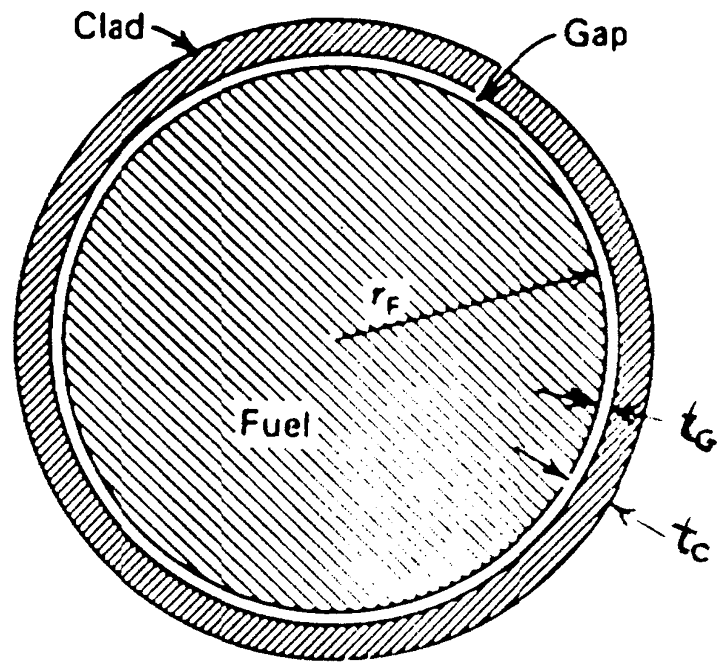


linear power density q' W/cm $= \int_0^{r_f} q''' 2\pi r dr = \pi r_f^2 q'''$ W/cm.

and $q' = \int_S q'' dA = 2\pi r_f q''$

$$\therefore\ 2\pi r_f q'' = \pi r_f^2 q''' \Rightarrow q'' = \frac{q''' r_f}{2}$$

Heat Conduction in the fuel meat



$$\rho c \cancel{\frac{\partial T}{\partial t}}^{\,0} = q''' + \nabla \cdot k \nabla T$$

$$\Rightarrow -\frac{1}{r}\frac{d}{dr}\left(k_F r \frac{dT}{dr}\right) = q'''(r) = q'''$$

assume uniform heat generation

$$\therefore -\int d\left(k_F r \frac{dT}{dr}\right) = \int r q''' dr$$

$$\Rightarrow -k_F \cancel{r} \frac{dT}{dr} = \frac{r^{\cancel{2}}}{2} q''' + \text{const.}$$

$= 0$ since $\frac{dT}{dr} = 0$ at $r = 0$

Now $k_F = f_n(T)$

$$\therefore -\int_{T_0}^{T_F} k_F(T)\, dT = \int_{r_0}^{r_F} \frac{r}{2} q''' dr = \frac{r_F^2}{4} q'''$$

$$\equiv -\bar{k}_F (T_F - T_0) \equiv \bar{k}_F \Delta T_{fuel}$$

average

$$\therefore \Delta T_{fuel} = \frac{r_F^2}{4\bar{k}_F} q''' = \frac{q'}{4\pi \bar{k}_F}$$

Note: $\Delta T_{fuel} \neq f_n(r)$

<u>Example</u>: UO_2 ceramic

$k_F \sim 0.02 - 0.03\ W/cm^2 K$

$\therefore \Delta T \sim 1400\,°C$

for q' of $500\ W/cm$.

Heat Transfer in the Gap

$$\rho c \cancel{\frac{\partial T}{\partial t}}_{0} = \cancel{q'''}_{0} + \nabla \cdot k \nabla T$$

$$\therefore \frac{1}{r}\frac{d}{dr}\left(k_G r \frac{dT}{dr}\right) = 0$$

$$\therefore k_G r \frac{dT}{dr} = \text{const}$$

$$= -q'/2\pi$$

$$\Rightarrow k_G \int_{T_F}^{T_C} dT = -\frac{q'}{2\pi}\int_{r_f}^{r_F+t_G} \frac{dr}{r}$$

$$\therefore k_G \Delta T_{Gap} \equiv k_G (T_F - T_c) = \frac{q'}{2\pi} \ln\left(\frac{r_F + t_G}{r_F}\right)$$

$$\therefore \Delta T_{Gap} = \frac{q'}{2\pi k_G} \ln\left(\frac{r_F + t_G}{r_F}\right) \simeq \frac{q'}{2\pi k_G} \frac{t_G}{r_F}$$

B.C.: heat flux known

ie $-k_G \frac{dT}{dr}\Big|_{r=r_F} = q'' = \frac{q'}{2\pi r_f}$

$\therefore k_G r_F \frac{dT}{dr}\Big|_{r=r_F} = -\frac{q'}{2\pi} = \text{const.}$

There is a problem with this:
k_G is variable + unknown
(depends on swelling, which depends on T, P, q''', history...)

$\therefore$ use effective HTC
(heat transfer coefficient):

$$h_G \Delta T_{Gap} = q''$$

$$\therefore \Delta T_{Gap} = \frac{q'}{2\pi r_F h_G} \sim \frac{500 \text{ W/cm}}{2\pi \times 0.5 \times 0.5}$$

$$< 300\,^\circ C$$

($h_G \sim 0.5 \rightarrow 1.1$ W/cm^{2}K)

Heat Transfer in the Clad

As in the gap:

$$\frac{1}{r}\frac{d}{dr}\left(k_c r \frac{dT}{dr}\right) = 0$$

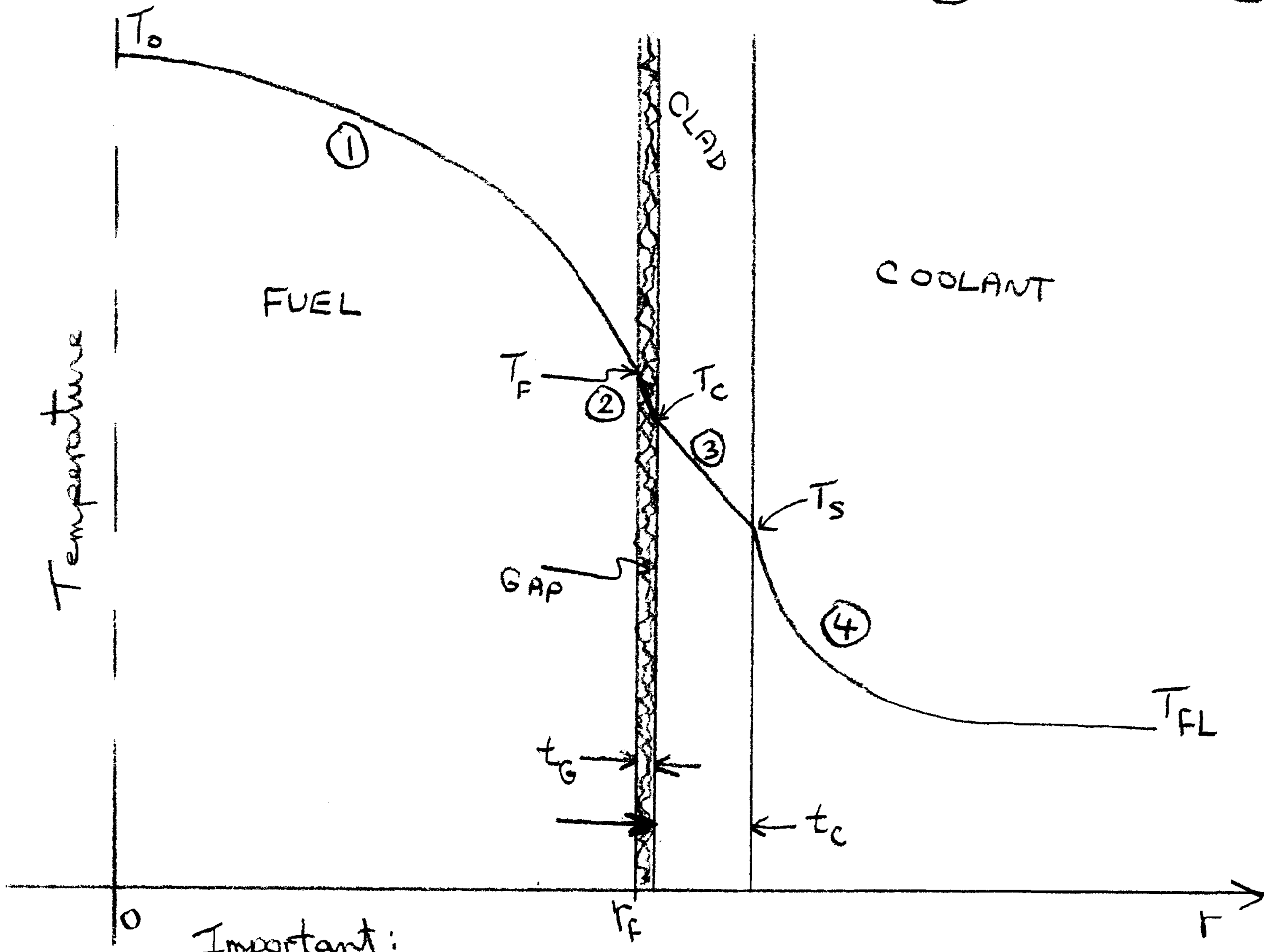
$$\Rightarrow \quad k_c \Delta T_{CLAD} \equiv k_c (T_c - T_s) = \frac{q'}{2\pi} \ln\left(\frac{r_f + t_G + t_c}{r_f + t_G}\right)$$

clad-coolant surface interface.

$$\therefore \Delta T_{CLAD} = \frac{q'}{2\pi k_c} \ln\left(\frac{r_f + t_G + t_c}{r_f + t_G}\right) \simeq \frac{q'}{2\pi k_c} \frac{(t_G + t_c)}{(r_f + t_G)}$$

$k_c \sim 0.11$ W/cm K $\Rightarrow \Delta T_{CLAD} \sim 80°C$ for $q' \sim 500$ W/cm.

$$T_0 = T_{FL} + \frac{q'}{2\pi}\left[\underset{1}{\frac{1}{2k_f}} + \underset{2}{\frac{1}{h_G r_f}} + \underset{3}{\frac{(t_G + t_c)}{k_c(r_f + t_G)}} + \underset{4}{\frac{1}{h_s(r_f + t_G + t_c)}}\right]$$



Important:
Temperature floats on coolant Temperature, T_{FL}