# *Spatial Kinetics (*CERBERUS Module)*

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## *Spatial Kinetics (*CERBERUS Module)*

- **Time-dependent problem in 3 dimensions and 2 energy groups**
- **Fast transients (e.g., LOCA arrested by SDS action)**
- **Delayed-neutron effects very important; assume G delayed-neutron precursor groups (typically G=6 or 15)**
- **Time–dependent neutron diffusion equation in two energy groups and three spatial dimensions (in matrix notation):**

$$\left(-M + F_p\right)\phi(\vec{r},t) + \sum_{g=1}^{G} \lambda_g C_g(\vec{r},t)\begin{pmatrix}1\\0\end{pmatrix} = \left(\frac{1}{v}\right)\frac{\partial \phi(\vec{r},t)}{\partial t} \qquad (8.1)$$

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**where,**

$$\phi(\vec{r},t) = \begin{pmatrix} \phi_1(\vec{r},t) \\ \phi_2(\vec{r},t) \end{pmatrix} \qquad (8.2)$$

$$\left(\frac{1}{v}\right) = \begin{pmatrix} \frac{1}{v_1} & 0 \\ 0 & \frac{1}{v_2} \end{pmatrix} \qquad (8.3)$$

## M is the leakage, absorption, and scattering matrix:

$$M = \begin{pmatrix} -\vec{\nabla}\cdot D_1\vec{\nabla} + \Sigma_{a1}(\vec{r},t) + \Sigma_{1\to 2}(\vec{r},t) & 0 \\ -\Sigma_{1\to 2}(\vec{r},t) & -\vec{\nabla}\cdot D_2\vec{\nabla} + \Sigma_{a2}(\vec{r},t) \end{pmatrix} \qquad (8.4)$$

# Spatial Kinetics (*CERBERUS Module)

$F_P$ is the prompt-production matrix:

$$F_p \equiv \left(1-\beta(\vec{r},t)\right)F_T = \begin{pmatrix} 0 & \dfrac{\left(1-\beta(\vec{r},t)\right)\nu\Sigma_f(\vec{r},t)}{k_0} \\ 0 & 0 \end{pmatrix} \tag{8.5}$$

and $\beta(\vec{r},t)$ is the total delayed fraction at position $(\vec{r},t)$:

$$\beta = \sum_{g=1}^{G} \beta_g \tag{8.6}$$

$C_g(\vec{r},t)$ = space-time concentration of group-g delayed-neutron precursor with decay constant $\lambda_g$. Satisfies balance equation

$$\frac{\partial}{\partial t}C_g(\vec{r},t) = \beta_g(r)\frac{\nu\Sigma_f(\vec{r},t)}{k_0}\phi_2(\vec{r},t) - \lambda_g C_g(\vec{r},t) \tag{8.7}$$

$k_0$ = initial multiplication constant of reactor (*not* related to time-dependent dynamic reactivity $\rho$)

# *Improved Quasi-Static (IQS) Method*

**CERBERUS based on IQS method. Flux factorized into space-independent amplitude *A* and space-and-time-dependent shape function $\psi$ :**

$$\phi(\vec{r},t) = A(t)\psi(\vec{r},t) \qquad \textbf{(8.8)}$$

**[Normalization A(0) = 1]**

**Most of time dependence cast into *amplitude* by demanding that an integral in the shape function be constant in time:**

$$\int\left[\frac{1}{v_1}\phi_1^*(\vec{r})\psi_1(\vec{r},t) + \frac{1}{v_2}\phi_2^*(\vec{r})\psi_2(\vec{r},t)\right]d\vec{r} = K \qquad \textbf{(8.10a)}$$

$\phi^*$ **= initial adjoint flux**

# *Improved Quasi-Static (IQS) Method*

**Substitute (8.8) into Eqs. (8.1) and (8.7) to get equations for shape $\psi$ and precursor concentrations $C_g$:**

$$\left(-M + F_p\right)\psi(\vec{r},t) + \frac{1}{A(t)}\sum_{g=1}^{G}\lambda_g C_g(\vec{r},t)\begin{pmatrix}1\\0\end{pmatrix} = \left(\frac{1}{v}\right)\frac{\dot{A}(t)}{A(t)}\psi(\vec{r},t) + \frac{\partial\psi}{\partial t} \qquad \textbf{(8.11)}$$

$$\frac{\partial}{\partial t}C_g(\vec{r},t) = \beta_g(\vec{r})\frac{\nu\Sigma_f(\vec{r},t)}{k_0}A(t)\psi_2(\vec{r},t) - \lambda_g C_g(\vec{r},t) \qquad \textbf{(8.12)}$$

**Eq. (8.11) is similar to time-independent equation, with extra terms in the amplitude and the precursor concentrations.**

# *Improved Quasi-Static (IQS) Method*

**Equation for amplitude obtained by integrating Eq. (8.11) - weighted by adjoint. Get point-kinetics-like equation:**

$$\dot{A}(t)=\frac{(\rho(t)-\beta_{eff})}{l^*(t)}A(t)+\frac{1}{K}\sum_{g=1}^{G}\lambda_g\eta_g(t) \qquad (8.14)$$

**where:**

$$\rho(t)=1-\frac{\left\langle\phi^*(\vec{r}),M\psi(\vec{r},t)\right\rangle}{\left\langle\phi^*(\vec{r}),F_T\psi(\vec{r},t)\right\rangle} \qquad (8.15)$$

**"Dynamic reactivity"**

$$=1-\frac{\text{losses}}{\text{production}}$$

**Neutron generation time:**

$$l^*(t)=\frac{K}{\left\langle\phi^*(\vec{r}),F_T\psi(\vec{r},t)\right\rangle} \qquad (8.16)$$

# Improved Quasi-Static (IQS) Method

**Effective total delayed fraction:**

$$\beta_{eff} \equiv \sum_{g=1}^{G} \beta_{g,eff} = \sum_{g=1}^{G} \frac{\left\langle \phi^*(\vec{r}), \beta_g F_T \psi(\vec{r},t) \right\rangle}{\left\langle \phi^*(\vec{r}), F_T \psi(\vec{r},t) \right\rangle} \qquad (8.17)$$

**and adjoint-weighted integrated precursors:**

$$\eta_g(t) = \int \phi_1^*(\vec{r}) C_g(\vec{r},t) d\vec{r} \qquad g = 1,\dots,G \qquad (8.18)$$

**which satisfy the balance equations**

$$\dot{\eta}_g(t) = K \frac{\beta_{g,eff} A(t)}{l^*(t)} - \lambda_g \eta_g(t) \qquad (8.19)$$

**We have a coupled system of equations for the shape, the amplitude, and the precursor concentrations: differential equations (8.11), (8.12), (8.14), and (8.19), together with integral equations (8.15) to (8.17).**

# *General Scheme of Solution*

Choose points in time, $t_0 = 0$, $t_1$, $t_2$,. . . at which shape function will be calculated. Intervals of 50-100 ms found appropriate for the first 2 or 3 seconds of LOCA transients. During SDS action, $t_j$ normally selected as, e.g., times when leading edge of shutoff rods coincides with model mesh lines. Following SDS action, larger intervals, up to several seconds, may be used. Solution follows recursively from each $t_j$ to $t_{j+1}$.

Starting point is solution to initial steady-state problem.

# *General Scheme of Solution (con't)*

At each subsequent time step the coupled set of equations is solved to find flux shape, amplitude, reactivity, precursors. The point-kinetics equations for the amplitude and integrated precursors are very quick to solve over a smaller time step. The shape equation requires most effort.

∴ A transient is solved as a sequence of flux-shape cases:

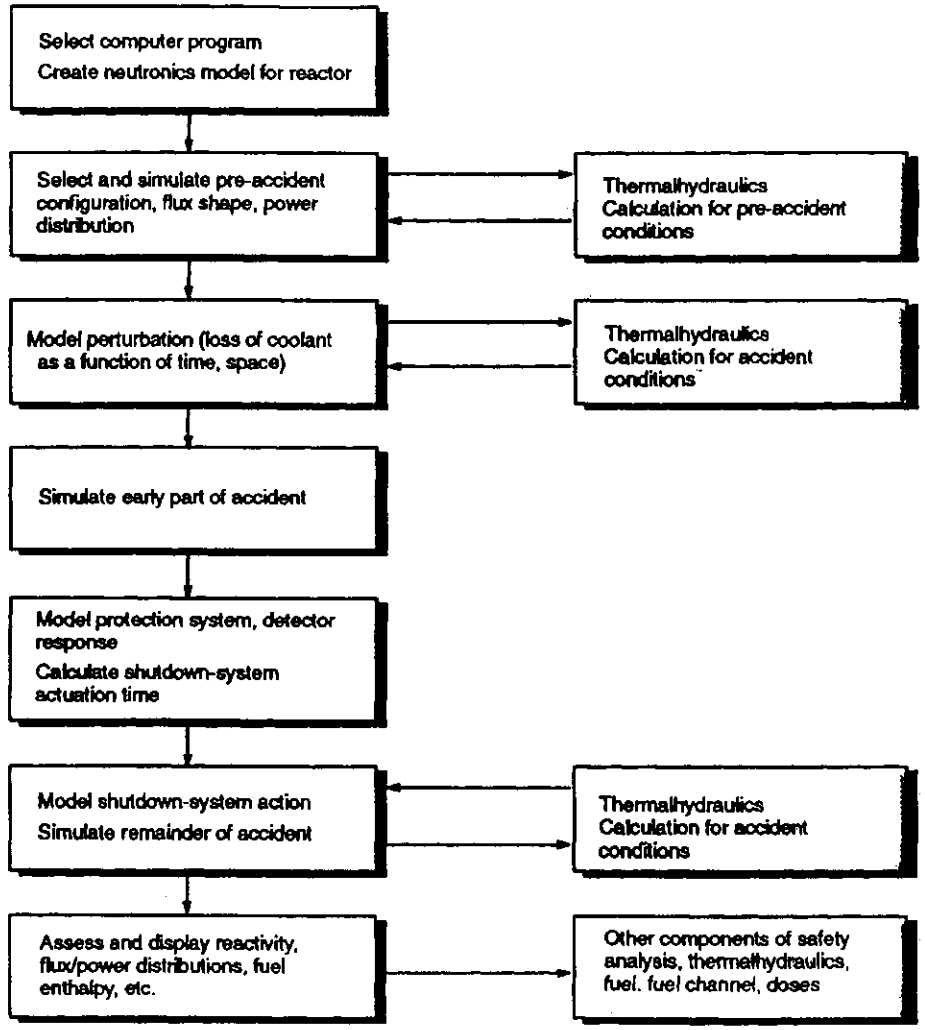
- Case 1 = initial steady state
- Case 2 = steady-state adjoint
- Cases 3 - ... = time-dependent cases

# General Scheme of Solution (con't)

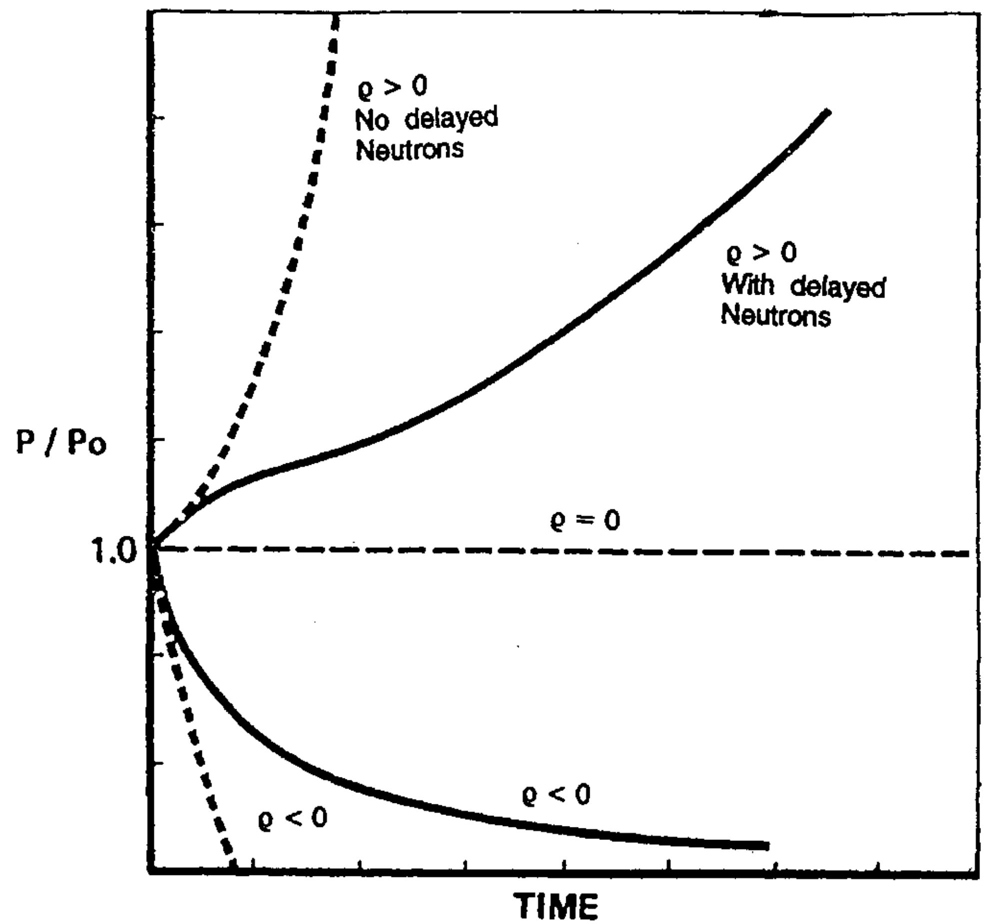
## Other features:

- The *CERBERUS module works in the *history-based* methodology of RFSP
- Capability to couple to thermalhydraulics calculation (e.g. CATHENA) - files exchanged at each flux-shape time step
- *TRIP-TIME module used to determine SDS actuation time.
- SDS *dynamic reactivity* more negative than *static reactivity* because precursors not in equilibrium with flux.

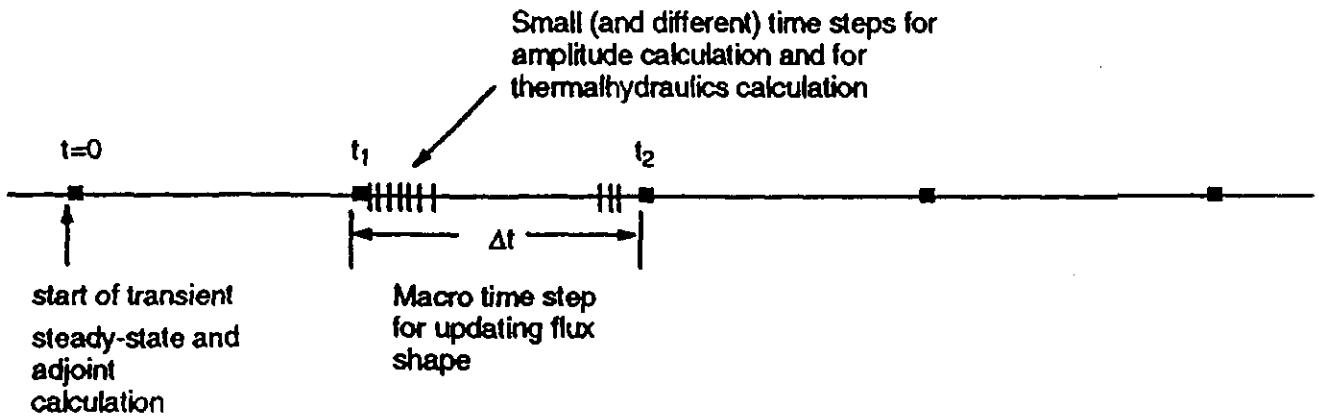
# Schematic of a Physics Analysis for a Large LOCA



# *Influence of Delayed Neutrons on Power Transients*



# Two-Tiered Numerical Computational Scheme



# *Example for the *DND Module*

*START  B. Arsenault
NUCIRC  DENSITIES & COOLANT TEMP
*MODEL  JUN93
*READ TAPEJUN93TA02
*DND
1.0 PPV
196902  ZTFUO1 NAT
80.        6.0
*CLOSE  NORMAL TERMINATION

# Example CERBERUS Case 1

*USED DAF STOREdc0

*START R.D. McArthur

DESIGN CENTER MODEL FOR PLNGS *MODEL

DESIGN CENTER TRANSIENT

*CEREBERUS PLGSUNIT1 140696610DES_CEN 1 THRMALCODEFIREBIRD

cd2opurity95.84 -

boroninmod0.263

the groups 8ch_loop1 des_cenf01

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| E | 2 | 4 | 2058400.0 | 0.95612 | 0.00002 | 1.5 | 300 |
| N | 6 | | | | | | |
| V | 7.648 E+060.2708E+06 | | | | | | |
| C | LZCRO1 | 0.637 | LZCRO2 | 0.466 | LZCR03 | 0.477 | |
| C | LZCRO4 | 0.510 | LZCRO5 | 0.504 | LZCR06 | 0.358 | |
| C | LZCRO7 | 0.364 | LZCRO8 | 0.470 | LZCR09 | 0.429 | |
| C | LZCR1O | 0.360 | LZCR11 | 0.560 | LZCR12 | 0.571 | |
| C | LZCR13 | 0.434 | LZCR14 | 0.399 | | | |

# Example *CERBERUS Case 1 (con't)

```
*delete     PHYS PARMS
*delete     FLUX/POWERPOWERS        BUNDLE
*delete     FLUX/POWERPOWERS        CHANNEL
*STORE
FROM SIMULDATA PLGSUNIT1 14034608714069661 0DE5_CEN DES_CEN 1PROMPTPDEC
TO  FLUX/POWERPOWERS BUNDLE
*STORE
FROM SIMULDATA PLGSUNIT1 14034608714069661 0DES_CEN DES_CEN 1CPROMPTPDC
TO  FLUX/POWERPOWERS CHANNEL
*print POWERS
*DELETE   DETECTORS
*DELETE   GEOMETRY NUMDETGRPS
*DELETE   GEOMETRY GROUPSPECS
*MAKE DAF STOREdc01
*CLOSE
```

# Example of *CERBERUS Case 2

*USE DAF STORE dc01
* START R.D. McArthur
DESIGN CENTRE MODEL FOR PLNGS

*MODEL DESIGN CENTER TRANSIENT

*CERBERUS PLGSUNIT1 140696610DES_CEN 2

cd2opurity95.84
boroninmod0.263

th groups 8ch_loop1 des cenf01

E 0 300

*DELETE PHYS PARMS

*PRNT MASS

*MAKE DAF STOREdc02

*CLOSE Normal Termination

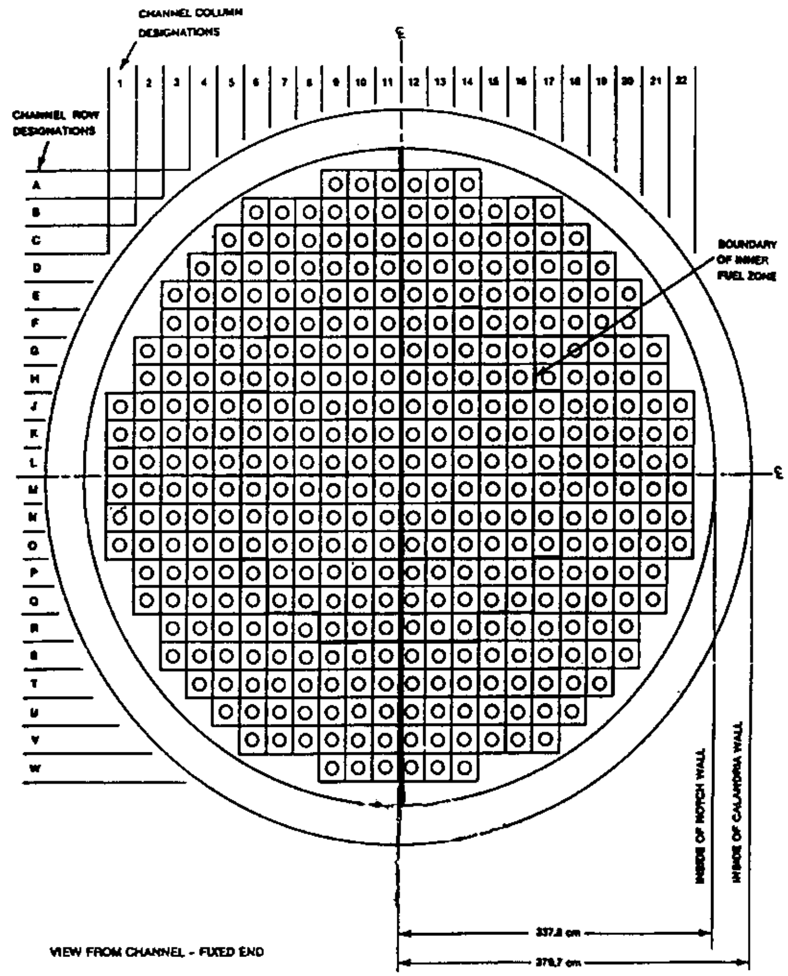
# Example *CERBERUS Case 3

```
*USE DAF STOREdc02
*START R.D. McArthur
DESIGN Center MODEL FOR PLNGS FPD2 844
* MODEL DESIGN CENTER TRANSIENT
*CERBERUS PLGSUNIT1 140696610DES_CEN 30.100
cd2opurity95.84
boroninmod0.263
th groups 8ch_loop 1 des_cenf03
E 2 4 2058400.0 0.95612 0.00050 1.5          300
* delete PHYS PARMS
*delete FLUX/POWERPOWERS BUNDLE
*delete FLUX/POWERPOWERS CHANNEL
FROM SIMULDATA PLGSUNIT1 140346087 1406966 IODES_CEN DES_CEN 3PROMPTPDEC
TO FLUX/POWERPOWERS BUNDLE
*STORE
FROM SIMULDATA PLGSUNIT1 140346087 140696610DES_CEN DES_CEN 3CPROMPTPD
TO FLUX/P OWERPOWERS CHANNEL
*print POWERS
*PRNT MASS
*MAKE DAF STOREdc03
*CLOSE NORMAL TERMINATION
```

# Example *CERBERUS Case 4

```
*USE DAF STOREdc03
*START RD. McArthur
DESIGN CENTER MODEL FOR PLNGS FPD2 844
*MODEL DESIGN CENTER TRANSIENT
*CERBERUS PLGSUNIT1 140696610DES_CEN 40.100
cd2opurity95.84
boroninmod0.263
th groups 8ch_loop1 des_cenf04
E 2 4 2058400.0 0.95612 0.00050 1.5                    300
*delete PHYS PARMS
* delete FLUX/POWERPOWERS BUNDLE
* delete FLUX/POWERPOWERS CHANNEL
*STORE
FROM SIMULDATA PLGSUNIT1 140346087 1406966 IODES_CEN DES_CEN 4PROMPTPDEC
TO FLUX/POWERPOWERS BUNDLE
*STORE
FROM SIMULDATA PLGSUNIT1 140346087 140696610DE5_CEN DES_CEN 4PROMPTPDC
TO FLUX/POWERPOWERS CHANNEL
*print POWERS
*PRNT MASS
*MAKE DAF STOREdcO4
*CLOSE NORMAL TERMINATION
```

# Subdivision of Heat-Transport-System Loops in CANDU 6



# Examples of Coolant Densities Calculated for the Various Channel Groups