Nuclear Theory - Course 227

NUCLEAR STRUCTURE

The Nucleus, Nuclear Particles

The atomic nucleus consists of *Z* protons and *N* neutrons, where *Z* and *N* are the *atomic number* and *neutron number* respectively. The total number of *nucleons* in the nucleus, that is, neutrons and protons, is equal to $Z + N = A$, where A is the *atomic mass number*.

A nuclear species with a given *Z* and a given *A* is called a *nuclide*. To distinguish a particular nuclide it is written in the form $_ZX^A$ where X is the chemical symbol for the element. Nuclides with the same *Z* but different *A* are called *isotopes*. Every element has a number of isotopes - most have both stable and unstable - some have only unstable which range from 3 (hydrogen) to 26 (tin), with an average of about 10 isotopes per element.

The mass of the proton is 1.67252×10^{-27} kg. It carries a positive charge of 1.60210×10^{-19} coulombs (C), equal in magnitude to the negative charge of the electron, and it is a stable particle.

The mass of the neutron is marginally greater than that of the proton, namely 1.67482×10^{-27} kg, and it is electrically neutral. The neutron is not stable unless it is bound in a nucleus. A free neutron decays to a proton with the emission of a β^- particle and an antineutrino, a process which has a half-life of 12 minutes. You will see later in this course that the average lifetime of neutrons in a reactor before they are absorbed or leak from the system is no greater than a millisecond. The instability of the neutron is therefore of no consequence in reactor theory.

Nuclear Masses

The mass of atoms are conveniently expressed in *unified mass units*, or u. The actual mass of a nucleus is measured on the *unified mass scale*, such that the mass of the C^{12} atom is precisely 12 u, and hence 1 u = 1.660438×10^{-27} kg.

The atomic mass of a nuclide should be distinguished from the chemical atomic weight which is the average weight of a large number of atoms of a given element. It is not quite the same as the mass of an individual atom unless the element contains a single isotope. Furthermore, you should note that the atomic weight unit on the *chemical scale* is defined as one-sixteenth of the average weight of an oxygen atom in a natural mixture of stable oxygen isotopes (0.204% O^{18}, 0.037% O^{17} and the rest O^{16}). In many calculations this slight distinction (about 3 ppm) is insignificant and the atomic mass, denoted by A, is used rather loosely.

Equivalence of Mass and Energy

Einstein showed that mass and energy are equivalent. The relationship between mass and energy changes may be written:

$$\Delta E = \Delta mc^2$$

where ΔE is the energy change expressed in joules, Δm is the accompanying change in mass given in kilograms and c is the velocity of light, equal to 3×10^8 meters per second.

A convenient and very common unit of energy in nuclear physics is the *electron volt* (abbreviated *eV*). It is the energy gained by an electron in being accelerated through a potential difference of 1 volt.

$$1 \text{ eV} = 1.6021 \times 10^{-19} \text{ joule}$$

$$1 \text{ keV} = 10^3 \text{ eV}$$

$$1 \text{ MeV} = 10^6 \text{ eV}$$

Using Einstein's formula it can readily be shown that converting 1 amu of mass yields ∿931 MeV of energy.

Binding Energy

The mass of the proton is 1.00728 u, and the mass of the neutron is 1.00867 u. The actual mass of a nuclide is not equal to the total mass of its individual nucleons, the difference being called the *mass defect*. This mass defect is a consequence of the equivalence of mass and energy and arises from the *binding energy* of the nuclide. This is the energy required to split the nuclide into its individual component nucleons. Experimental results (Figure 1) show that except for a few light nuclides, the binding energy per nucleon in the nucleus, increases rapidly as the size of the nucleus increases up to about A = 60, but for greater values it decreases again gradually. This means that nuclei of intermediate mass are more strongly bound than the light and the heavy nuclei. Thus energy may be released by combining two light nuclei (*fusion*);

$$\begin{array}{ccccccccc} {}_1H^2 & + & {}_1H^3 & \longrightarrow & {}_2He^4 & + & {}_0n^1 & \\ 2.0147 & & 3.0169 & & 4.0039 & & 1.0087 & \text{masses (u)} \end{array}$$

Here 0.019 u is converted to 17.7 MeV.

Or by splitting a heavy nucleus into two nuclei of intermediate mass (fission);

$$\begin{array}{llcllll} {}_{92}U^{235} & + \; {}_{0}n^{1} & \longrightarrow & {}_{36}Kr^{95} & + \; {}_{56}Ba^{139} & + \; 2\,{}_{0}n^{1} & \\ 235.044 & 1.009 & & 94.903 & 138.918 & 2.017 & \text{Mass (u)} \end{array}$$

Here 0.215 u is converted to 200.2 MeV.

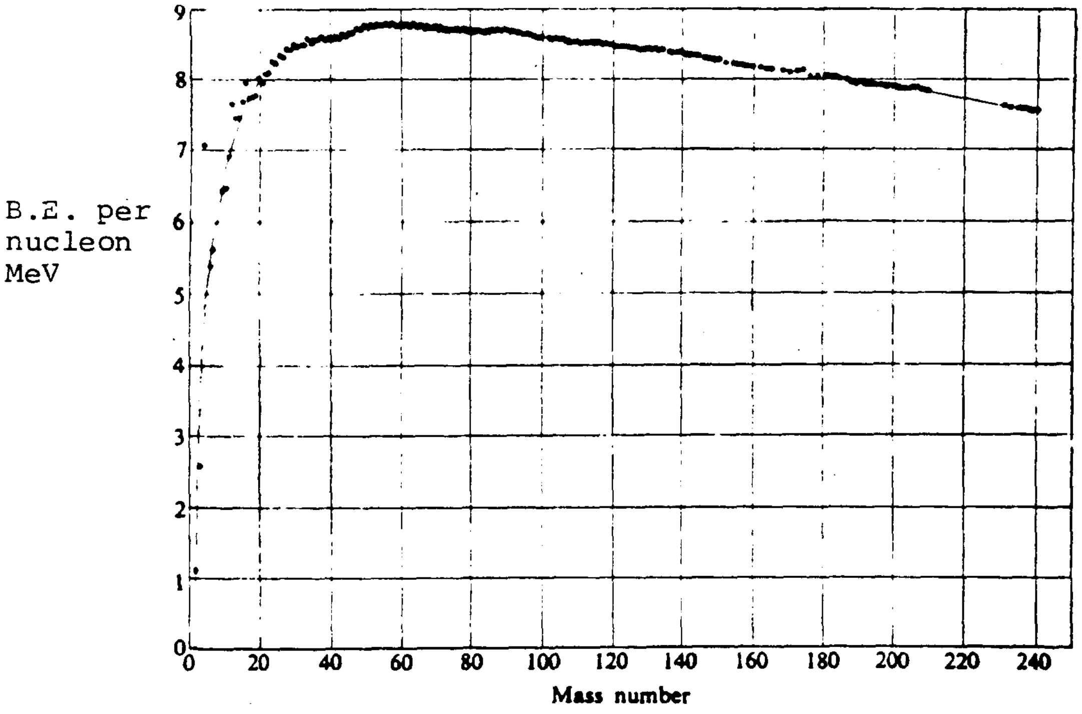

Figure 1

Binding Energy vs Mass Number

Nuclear Forces

Between two electric charges of the same sign there is a repulsive force which is called a *Coulomb force*. Since nuclei may contain a large number of positive protons each repelling the other due to Coulomb forces it is clear that there must be other forces present which are attractive. These are short range *nuclear forces*. They act between all adjacent nucleons, whether n-p, n-n, or p-p, and drop off rapidly on separation of the nucleons.

The lighter stable nuclei contain roughly equal numbers of neutrons and protons (eg, ${}_6C^{12}$, ${}_8O^{16}$, ${}_9F^{19}$, ${}_{11}Na^{23}$). As the number of protons in the nucleus increases, the long range Coulomb forces build up more rapidly than the nuclear forces which only have short range. Therefore, in order for heavier nuclei to remain intact more neutrons are required to supply binding forces between all particles to overcome the distruptive Coulomb forces. As a result, the n/p ratio required for stability gradually increases from one in light nuclei to about one and a half in heavier nuclei. This increase in the n/p ratio for stable nuclei is shown in Figure 2.

It should be noted that this is a very simple model and it cannot explain all the facts of nuclear stability or decay.

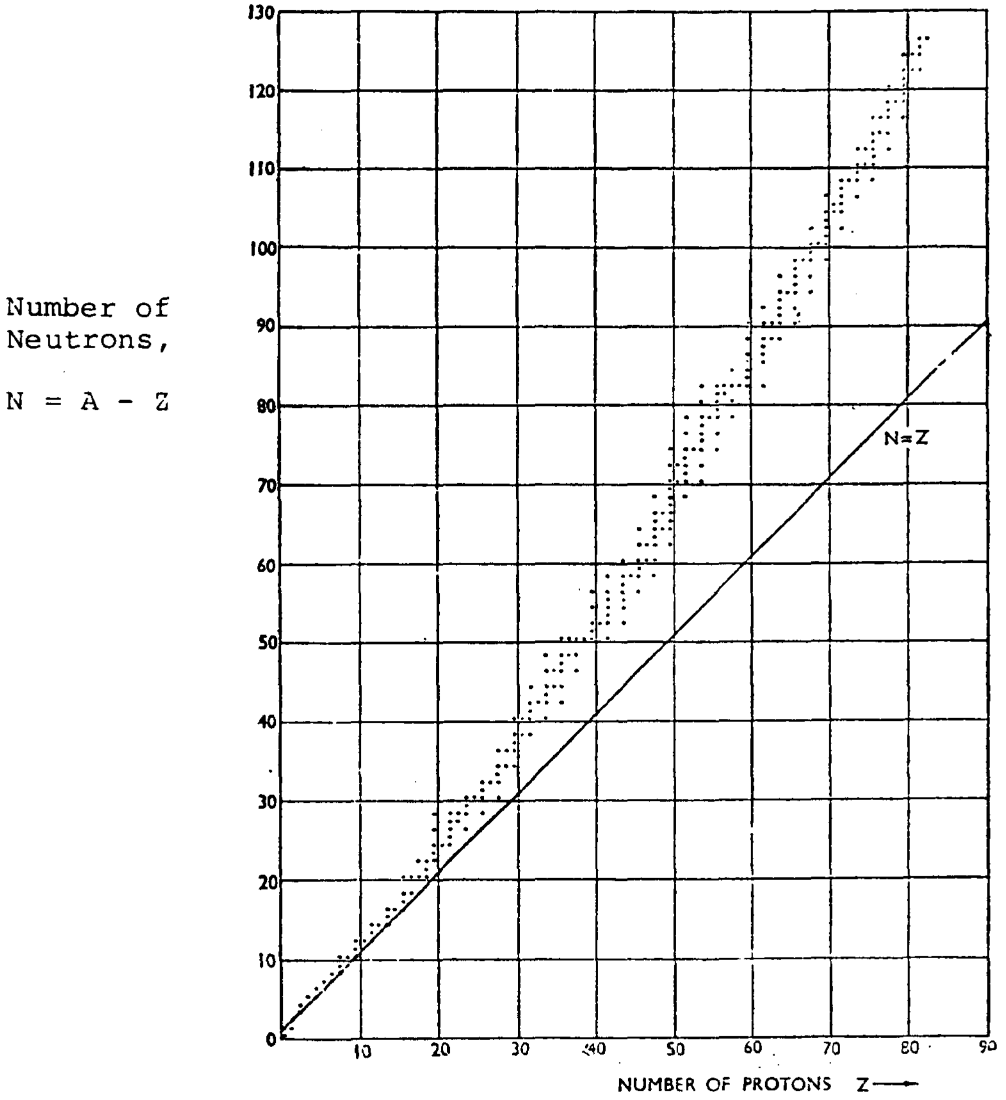

Figure 2

Neutron/Proton Ratio

For reasons of no particular significance to us, there is a limit to the number of excess neutrons a nucleus can live with, and as a result the heavy nuclei are all unstable and there are no naturally occurring elements having a value of A greater than 238.

Nuclear Energy Levels

A nucleus is said to be in its *ground state* when the nucleons are arranged in such a way that the potential energy is a minimum. If it is not in its ground state it is said to be in an *excited state* and the excess of energy is called *excitation energy*. The potential energy does not take on a continuous range of values, but has discrete values which are termed *energy levels*. For heavy nuclei these energy levels have a minimum separation of about 0.1 MeV, for light nuclei this separation is much greater.

Radioactivity

All the naturally occurring nuclides heavier than lead ($Z = 82$) and a few lighter nuclides are unstable and are *naturally radioactive*. They decay by emitting either an *alpha particle* (helium nucleus) or a *beta particle* (fast electron). In most cases the resulting nucleus, or *daughter*, is produced in an excited state. It then decays to its ground state by the emission of one or more *gamma photons*. Usually, but not always, this occurs instantaneously, ie, within 10^{-14} seconds of the formation of the daughter.

Radioactivity is governed by only one fundamental law, namely that the probability of a radionuclide decaying per unit time is constant and independent of external conditions. This constant is called the *decay constant* and is denoted by λ.

Thus the rate of change of single kind or radionuclide is:

$$\frac{dN}{dt} = -\lambda N$$

where: N = Number Density in ATOMS/cm^3
λ = decay constant in 1/s

The solution to this simple differential equation is:

$$N(t) = N_0 e^{-\lambda t}$$

The time for the number of atoms to be diminished to one half of its original value is called the *half-life* ($t_{\frac{1}{2}}$).

$$N(t) = \tfrac{1}{2}N = N_0 e^{-\lambda(t_{\frac{1}{2}})}$$

Thus:

$$\tfrac{1}{2} = e^{-\lambda t_{\frac{1}{2}}}$$

$$\ell n \tfrac{1}{2} = -\lambda t_{\frac{1}{2}}$$

$$t_{\frac{1}{2}} = \frac{0.693}{\lambda}$$

The *activity* of a sample is simply the number of disintegrations per unit time or N. The historic unit for activity is the *Curie* (Ci), which is 3.7×10^{10} disintegrations per second (dps). The SI unit for activity is the *Becquerel* (Bq).

$$1 \text{ Bq} = 1 \text{ dps}$$

ASSIGNMENT

1. Calculate the mass defect and the binding energy for ${}_6C^{13}$.

2. In your own words, explain binding energy.

3. Xenon-135 has a half-life of 9.16 hours. What is its decay constant?

4. Sketch a graph of activity versus time, in half lives, for a radionuclide assuming that the activity is A_0 at time zero.

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