Chapter 7 Empirical Correlations

7.1 Introduction

7.1.1 Chapter Overview

As previously discussed, supporting relations are required to provide the necessary information for the conservation and state equations. This chapter provides the needed correlations, albeit in a very limited sense. The intent here is just to provide a glimpse of the huge number of hydraulic and heat transfer correlations available in the open literature.

7.1.2 Learning Outcomes

Objective 7.1	The student should be able to apply typical correlations to the thermalhydraulic models developed in earlier chapters.					
Condition	Workshop or project based investigation.					
Standard	75%.					
Related concept(s)	Typical correlations.					
Classification	Knowledge	Comprehension	Application	Analysis	Synthesis	Evaluation
Weight	a	a	a			

7.1.3 Chapter Layout

The chapter first discusses the relationship between void fraction and quality, then discusses friction factors and heat transfer. Next some alternate sources of water properties are given, followed by a brief look at flow regimes and models for valves and pumps. Finally a correlation for critical heat flux is presented. Please note that when it comes to thermalhydraulic correlations, it is user beware. It is unlikely that the correlations are valid much beyond the range of operating conditions that the correlations were developed for. The user is advised to check the range of applicability carefully before relying on any correlation that is critical to the investigation at hand.

7.2 Empirical Correlations

The primary areas where support is needed are:
1) relationship between quality and void fractions, i.e., slip velocities in two phase flow (to link the mass and energy conservation equations via the state equation);
2) the stress tensor, τ (effects of wall shear, turbulence, flow regime and fluid properties on momentum or, in a word: friction);

3) heat transfer coefficients (to give the heat energy transfer for a given temperature distribution in heat exchangers, including steam generators and reactors);

4) thermodynamic properties for the equation of state;

5) flow regime maps to guide the selection of empirical correlations appropriate to the flow regime in question;

6) special component data for pumps, valves, steam drums, pressurizers, bleed or degasser condensors, etc; and

7) critical heat flux information (this is not needed for the solution of the process equations but a measure of engineering limits is needed to guide the use of the solutions of the process equations as applied to process design;

The above list of correlations, large enough in its own right, is but a subset of the full list that would be required were it not for a number of key simplifying assumptions made in the derivation of the basic equations. The three major assumptions made for the primary heat transport system are:

1) one dimensional flow;

2) thermal equilibrium (except for the pressurizer under insurge); and

3) one fluid model (i.e. mixture equations).

These are required because of state of the art limitations. References [BAN80, BER81, CHO74, COL72, CRA57, GIN81, HSU76, ITT73, IDE60, LAH77, MIL71, STE48, TON65] are recommended for further reading.

7.3 Two Phase Flow Void Correlations

In a two-phase flow situation, the relationship between the volume fraction occupied by the less dense phase (the void fraction, α) is related to the weight fraction occupied by the less dense phase (quality, x). The relationship depends on the densities of each phase <u>and</u> on the velocities of each phase. For instance, if both phases are moving with the same velocity, then the relationship is simplified and depends on densities only. This is the homogeneous model. In this case the quality or weight fraction, occupied by a void, α, is simply the density weighted void fraction:

$$\frac{\text{wt. of steam}}{\text{wt. of steam+liquid}} \equiv x = \frac{\rho_g \alpha}{\rho_g \alpha + \rho_f (1-\alpha)} \tag{1}$$

Conversely, the void fraction is simply the volume weighted mass fraction:

$$\alpha = \frac{v_g}{(1-x)v_f + x v_g} \tag{2}$$

Equation 1 or 2 can be rearranged to give:

$$\left(\frac{x}{1-x}\right) \left(\frac{1-\alpha}{\alpha}\right) \frac{\rho_f}{\rho_g} = 1 \tag{3}$$

which is a commonly used form of the homogeneous model. Figure 7.1 shows a plot of α vs. x for various pressures for heavy water.

To account for non-homogeneous conditions (as dictated by the flow regime map), various attempts have been made. Armand gives:

$$\alpha = \frac{(0.833 + 0.167x)}{(1-x) \, v_f + x v_g} \, x v_g \tag{4}$$

where v_v and v_l are the specific volumes of vapour and liquid respectively ($v = 1/\rho$).

Other typical correlations are given in reference [CHA77b].

Returning to the homogeneous model, we note that the relation is highly non-linear. By taking the derivative we obtain:

$$\frac{\partial \alpha}{\partial x} = \frac{1}{x \left(1 - \frac{\rho_v}{\rho_l}\right) + \frac{\rho_v}{\rho_l}} - \frac{x / \left(1 - \rho_v / \rho_l\right)}{\left[x + \rho_v / \rho_l - \rho_v\right]^2} \tag{5}$$

A plot is shown in Figure 7.2. We see that the change in void for a change in quality is the largest at small void and quality fractions. The first steam bubble formed gives the largest increase in void fraction. The onset of boiling and void collapse would be expected, then, to give the largest associated pressure changes. This often proves difficult to handle numerically and often significantly adds to the dynamic behaviour of the primary heat transport system.

7.4   Friction

The classical form for pressure drop in straight pipes is based on Darcy [CHA77b].

$$\Delta P_f = \frac{f \, L}{D} \left(\frac{W}{A}\right)^2 \frac{1}{2 \rho g_c}, \tag{6}$$

where f is given by the Moody diagram or by the Colebrook equation:

$$\frac{1}{\sqrt{f}} = 0.86858 \, \ln \left[\frac{1}{3.7} \frac{\varepsilon}{D} + \frac{2.51}{Re \sqrt{f}}\right]. \tag{7}$$

The ratio ε/D is the relative roughness and Re is the Reynold's number. For smooth pipes, this can be simplified to:

$$f = 0.0056 + \frac{0.50}{Re^{0.32}} \tag{8}$$

When in two phase flow, the general approach is to modify the single-phase pressure drop by a two-phase multiplier. This multiplier is a function of, at least, quality. Other parameters are used, depending on the investigator. A simple correlation is that used for homogeneous turbulent flow. This multiplier by Owens gives:

$$\Delta P_f = \frac{f \, L}{D} \left(\frac{W}{A}\right)^2 \frac{1}{2 \rho g_c} \phi^2 \tag{9}$$

where

$$\phi^2 = 1 + x \left(\frac{\rho_f}{\rho_g} - 1 \right) \tag{10}$$

Additional frictional losses, over and above the losses in straight pipes, occur when the fluid changes direction. This occurs in expansions, contractions and bends. Equation 6 is modified to

$$\Delta P_f = \left(\frac{f\,L}{D} + k \right) \left(\frac{W}{A} \right)^2 \frac{1}{2\rho g_c} \tag{11}$$

and k is evaluated from standard published data such as Crane [CRA57], ITT Grinnell [ITT73], Idel'Chik [IDE60] or Millar [MIL71].

7.5 Heat Transfer Coefficients

Heat transfer can be convective, conductive or radiative. Radiation plays only a minor role in primary heat transport except under extreme accident conditions not within the scope of this course. Thus radiation is neglected. Conduction is straightforward and is adequately covered by the standard texts. Fourier's Law governs for energy transfer in solids (pressure tubes, boiler tubes, etc.). Convective heat transfer is, however, very dependent on flow regime. Specific correlations exist for the full range from single phase laminar flow to two phase turbulent flow. Specification of the flow regime is self evident in some cases. For instance, a specific correlation exists for the condensing section of a steam generator. [CHA77b] covers typical correlations. All of these correlations result in an estimate of the convective heat transfer coefficient, h_N. This coefficient is not to be confused with the enthalpy, h.

An illustration of the function relationships typically found in correlations for h_N, Dittes Boelter gives:

$$h_N = \left(\frac{k}{D_e} \right) (0.023) \left(\frac{W D_e}{A\mu} \right)^{0.8} \left(\frac{\mu C_p}{k} \right)^{0.4} \tag{12}$$

where
  D_e = hydraulic diameter = 4 flow area / wetted perimeter,
  k = thermal conductivity,
  μ = dynamic viscosity of fluid,
  C_p = specific heat of fluid.

7.6 Thermodynamic Properties

It seems that each and every major simulation code uses a unique blend of water properties obtained from various sources. For instance, the SOPHT code [CHA77b] uses the following tables of saturation properties and their partial differentials at constant density derived from steam tables from which the properties of subcooled liquid and superheated steam can be calculated. This scheme and its accuracy were reported by Murphy et al. [MUR68]. The SOPHT property tables were derived from data of the following reports.
  1)   ASME steam tables, 1973 (Ontario Hydro steam table program).
  2)   "Specific Heat and Enthalpy of Liquid Heavy Water", by P.Z. Rosta, TDAI-43 AECL,

	November 1971.
3)	Tables of the Thermodynamic Properties of Heavy Water, by J.N. Elliott, AECL-1673, 1963.
4)	"The Density of Heavy Water", by S.L. Rivkin, Reactor Science and Technology, Volume 14, 1961.
5)	"Heavy Water Thermophysical Properties", by V.A. Kirillin, et al, 1963.

The thermal conductivity, viscosity, surface tension of saturated, natural and heavy water are prepared as functions of temperature or pressure. The data are found in steam tables and:

1)	"An Experimental Investigation of the Viscosity of Heavy Water Vapour at Temperatures of 100 to 500 °C and Pressures of 0.08 to 1.3 bar', by D.L. Timrot, et al, Teploenergetika, 1974, and
2)	"Some Physical Properties of Heavy Water", by D.G. Martin, Ontario Hydro, CCD-72-5.

Other sources of water properties are:

1)	STPH - A new Package of Steam-Water Property Routines, WNRE 467.
2)	Properties of Light Water and Steam from 1967 ASME Steam Tables as Computer Subroutines in APEX IV and Fortran-IV, CRNL 362.
3)	Thermodynamic Properties, Gradients and Functions for Saturated Steam and Water, AECL 5910.
4)	Tables of Transport Properties for Water and Steam in S.I. Units, WNRE 171.
5)	ASME Steam - Properties - A Computer Program for Calculating the Thermal and Transport Properties of Water and Steam in SI Units, WNRE 182.

For this course, light water properties are supplied on disk (with documentation) with these notes. This has been discussed in some detail in chapter 4 and appendix 4. Approximate D_2O curve fits can be found in [FIR84].

7.7   Flow Regime Maps

Flow regime modelling represents one area on the forefront of investigation. Some maps do exist but large gray transition areas exist [BER81, HSU76 and CHO74]. One such map is given by Figure 7.3. The design point for typical HT flow is also shown???. We see that the flow is well into the homogeneous region.

In general, for a first estimate for calculations at nominal design conditions for the Heat Transport system and auxiliaries, homogeneous conditions can be assumed. A check on the flow regime must be made, however, for detailed calculations. Special care must be taken during transients since new flow regimes will likely be established under low flow, low pressure or high quality/void fraction conditions.

7.8   Special Component Data

[CHA77b] gives a good discussion on special component modelling. Hence, the salient features only will be presented here.

7.8.1   Reactor Channel Heat Flux

The fundamental neutron flux is a cosine distribution in the axial direction. However, because of non-uniformity (discrete fuel bundles, control rods, etc.), the neutron flux is quite distorted from this fundamental shape. Radial flattening is deliberate in order to get better utilization of the fuel from a burnup point of view and from a thermal performance point of view. It is inefficient to have the radially peripheral channels operating at lower heat fluxes than the central channels. Typical distributions are shown in figure 7.4. The power distribution follows the neutron flux distribution, to a first order approximation. This power or energy generation can be directly input into the basic energy conservation equation.

7.8.2   Pumps

Herein, the characteristics of a pump are described insofar as they affect the process system under normal conditions. No attempt will be made to describe abnormal conditions, such as two phase flow in pumps which could occur during accident situations. No attempt will be made to discuss the design of a pump. These areas are subjects onto themselves. The discussion herein follows [CHA77b].

From the point of view of modelling a pump in a process system simulation, it is necessary to know the head or ΔP developed by the pump. This is a function of flow through the pump. Stepanoff [STE48] gives the complete head-flow characteristics as shown in figure 7.5. According to Bordelon [CHA77b] and Farman [CHA77b], these complete characteristics can be simplified using the pump affinity laws (assuming constant efficiency at points 1 and 2):

$$\frac{Q_1}{Q_2} = \frac{n_1}{n_2}, \quad \frac{H_1}{H_2} \approx \left(\frac{n_1}{n_2}\right)^2, \quad \frac{(BHP)_1}{(BHP)_2} = \left(\frac{n_1}{n_2}\right)^3 \tag{13}$$

where Q = flow
    H = head
    n = speed
    BHP = brake horsepower.

The resulting simplified curve is shown in figure 7.6. A similar process gives the simplified torque curve as shown in figure 7.7.

Curve fits are used to reduce these curves to algebraic expression form, suitable for computer codes. Usually the simplified curves obtained from the Stepanoff curves have to be adjusted using test data in order to represent the actual pump characteristics as accurately as possible.

Models exist for modifying the curves to account for two-phase flow but the reader should be aware of the extreme difficulty of modelling two-phase flow in pumps and to use these very approximate models with the appropriate reservations.

One final mode of pump operation to note is the pump rundown due to loss of power and the subsequent braking of the pump impeller. [CHA77b] covers the pump rundown equation and uses a loss coefficient, k, for the braked pump since it is just a hydraulic impedance in this case. This is similar to the k for pipe bend losses in the (fl/d+k) term of the momentum equation.

7.8.3   Valves

Proper modelling of valves are important so that their hydraulic resistance, and hence flow, can be correctly calculated as a function of pressure drop across the valve. Special emphasis should be given for relief and safety valves because of their importance in determining primary heat transport transient behaviour.

Again, [CHA77b] covers flow through valves in some detail. To provide some feeling of the important parameters, a summary is provided here.

For single phase liquid, when no choking occurs in the valve.

$$\Delta P_{VALVE} < k_m \left(P - r_c P_v \right) \tag{14}$$

we have:

$$\Delta P_{VALVE} = k_{cv} \left(\frac{W}{A} \right)^2 \frac{1}{2\rho g_C} \tag{15}$$

$$k_{cv} = \frac{2A^2}{A_{cv}^2 C_v^2 \rho_w} \tag{16}$$

where k_m = valve recovery coefficient (typically 0.6)

P = upstream pressure,

P_c = critical pressure of fluid,

P_v = vapour pressure at flow temperature,

r_c = function of P_v/P_c (see figure 7.8),

k_{cv} = effective resistance coefficient,

A = pipe area (m^2),

A_{cv} = fraction valve opening,

C_v = valve coefficient ($m^3 s\sqrt{kPa}$), obtainable from the valve manufacturer,

ρ_w = density of water at 20°C and atmospheric pressure = 1000 kg/m³.

For flashing conditions, when

$$\Delta P_{VALVE} > k_m \left(P - r_c P_v \right) \tag{17}$$

the flow is given by:

$$W = A_{cv} C_v \rho \sqrt{k_m (p - r_c p_v) \frac{\rho_w}{\rho}} \tag{18}$$

where ρ is the upstream density (figure 7.8??). Note that flow is independent of downstream pressure for choked flow.

For steam flow, the rated capacity at rated conditions are usually known, i.e., we have W_{rated} at A_{cv}, P_{rated} and T_{rated}. Choked conditions usually prevail. Flow at other P & T's are obtained by prorating via:

$$W(P,T) = W_{RATED} \frac{P}{P_{RATED}} \frac{\sqrt{T_{RATED}}}{T}$$

$$= W_{RATED} \sqrt{\frac{P\rho}{P_{RATED} \, \rho_{RATED}}} \quad \text{if} \quad \frac{P}{\rho} = RT \tag{19}$$

T is absolute temperature.

Various valve manufacturers may have their own formulas for calculating capacity. Valves also may have non-linear relationships for stem stroke vs. opening. Figure 7.9 shows some typical types.

7.8.4   Critical Heat Flux, CHF

As a fuel bundle power is raised, the closer the bundle comes to a heat transfer crisis. This critical heat flux represents an upper bound for design purposes. Figure 7.10 shows typical CHF data as a function of quality. Implicit in this data is a functional dependence on pressure and flow. For low power, high quality situations (as might occur under low flow conditions), dryout of the fuel pencil surface occurs. The surface temperature rises dramatically as illustrated in figure 7.11 and the fuel sheath may melt, causing fuel failure. For high power, low quality situations (as might occur for over-power conditions), fuel centerline melting occurs first, before surface dryout. In that event, fission product release in the event of a fuel pin rupture would be large.

Thus, for economic and safety reasons a lower bound CHF correlation was chosen below all data points for 37 CANDU element fuel.

$$CHF = 0.57 \times 2{,}798.76 \; e^{(-3.2819x)} \quad kw/m^2 \tag{20}$$

This correlation is constantly under scrutiny and revision. It has served a useful purpose, however, for a number of years, dating back to Bruce A design. Testing at CRL attempts to account for non-uniform heat flux in segmented bundles.

A typical channel power and channel quality profile is given in figure 7.12. If replotted as power vs. quality, figure 7.13 results. Superimposed on the CHF curve (figure 7.14), we see the relationship between the operating heat flux and the critical heat flux. To calculate the reserve, the power-quality curves are calculated for increasingly higher powers until the power-quality curve touches the CHF-curve (as shown by the dotted line in figure 7.14). This power, then, is the maximum power achievable without dryout or melting. The ratio of this maximum power to the nominal operating power is the critical power ration, CPR.

Typical reserve margins (to allow for instrumentation error, simulation of neutron flux error, refuelling rupple, and operating margin) requires a CPR of 1.35 to 1.40. The heat transport design conditions are set, in part, based on this restraint.

7.9   Exercises

1.   Add coding for valves, pumps, heat transfer coefficients and friction to your system code.
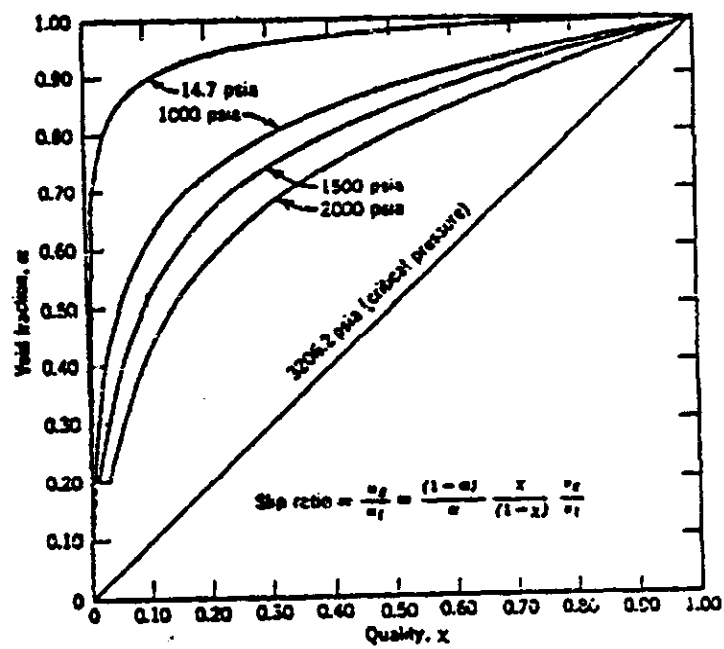
Figure 7.1 Void fraction versus quality for mixtures of saturated liquid and vapour water.
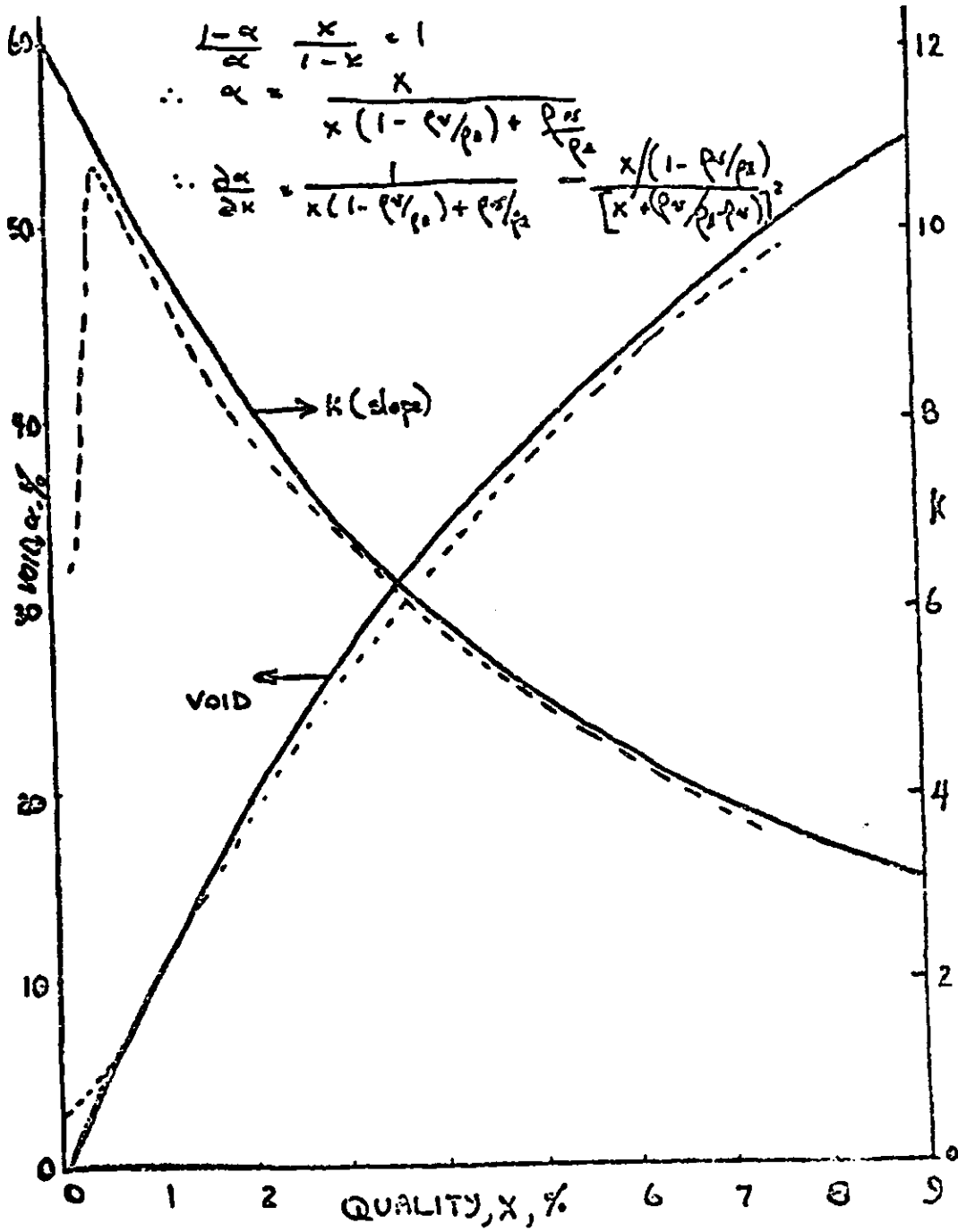
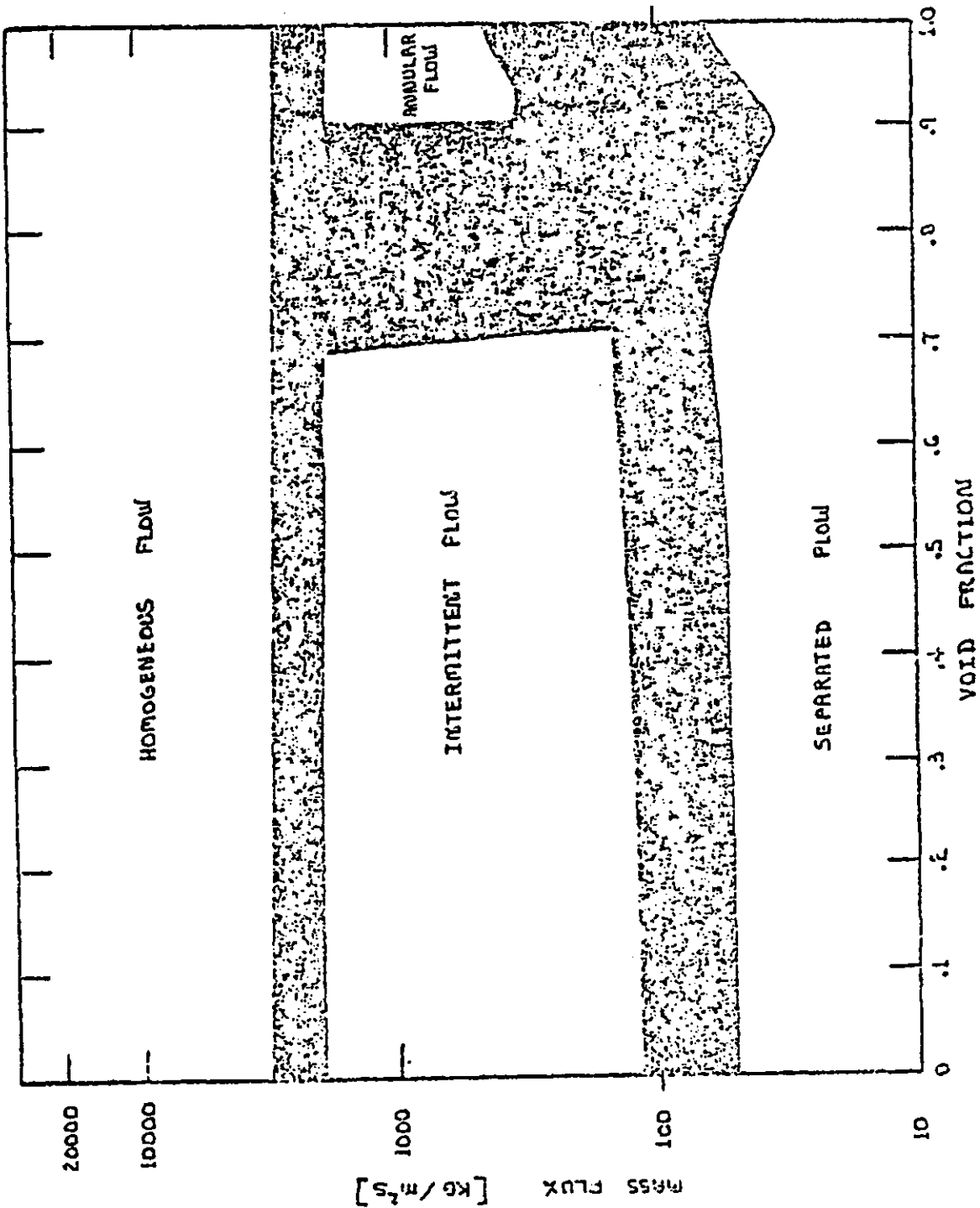**Figure 7.2** α versus x and $\partial\alpha/\partial x$.

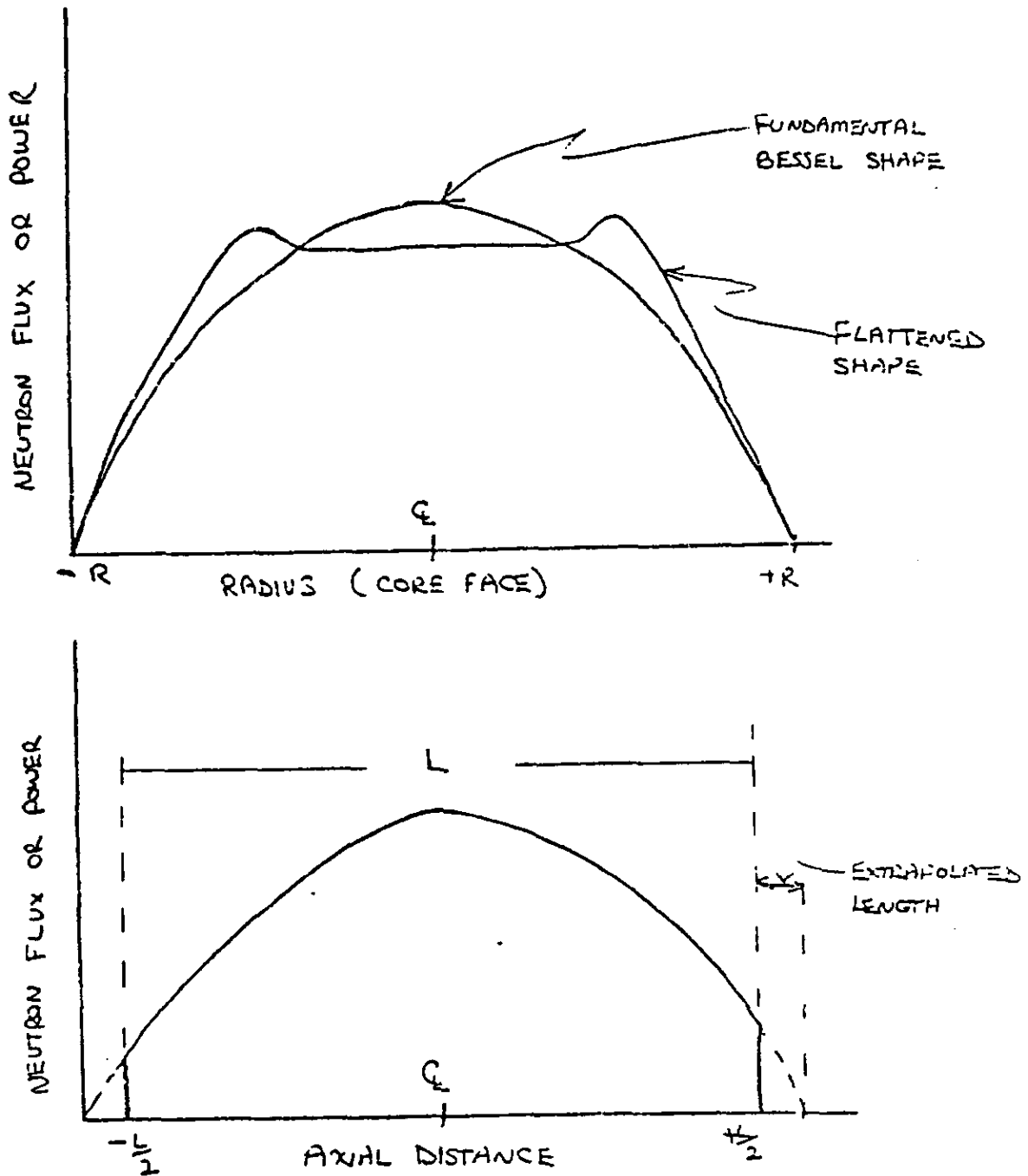**Figure 7.3** Flow regimes in horizontal pipes.
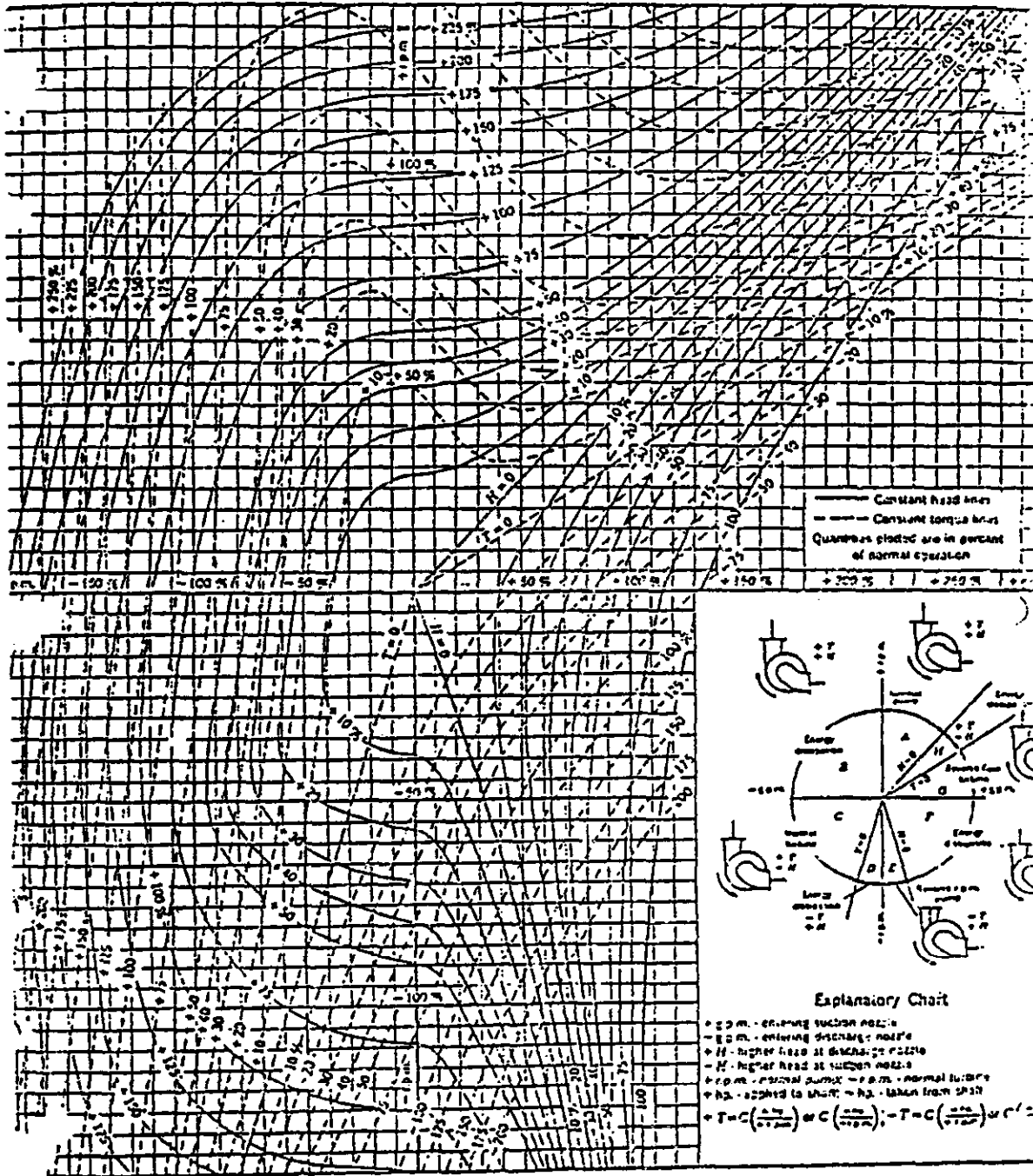
Figure 7.4 Typical power distributions.

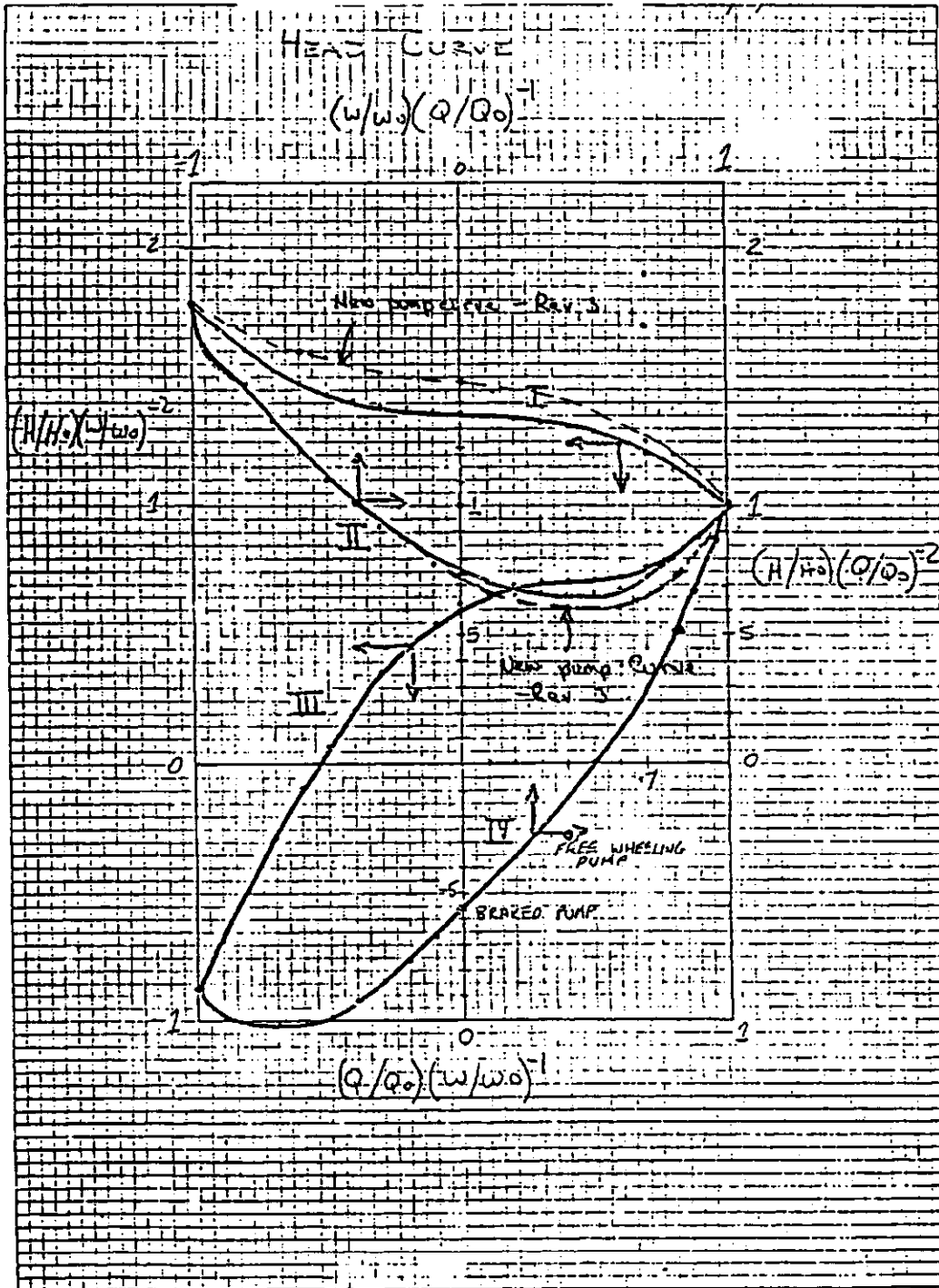**Figure 7.5** complete pump characteristics, double-suction pump, speed = 1800 rpm.

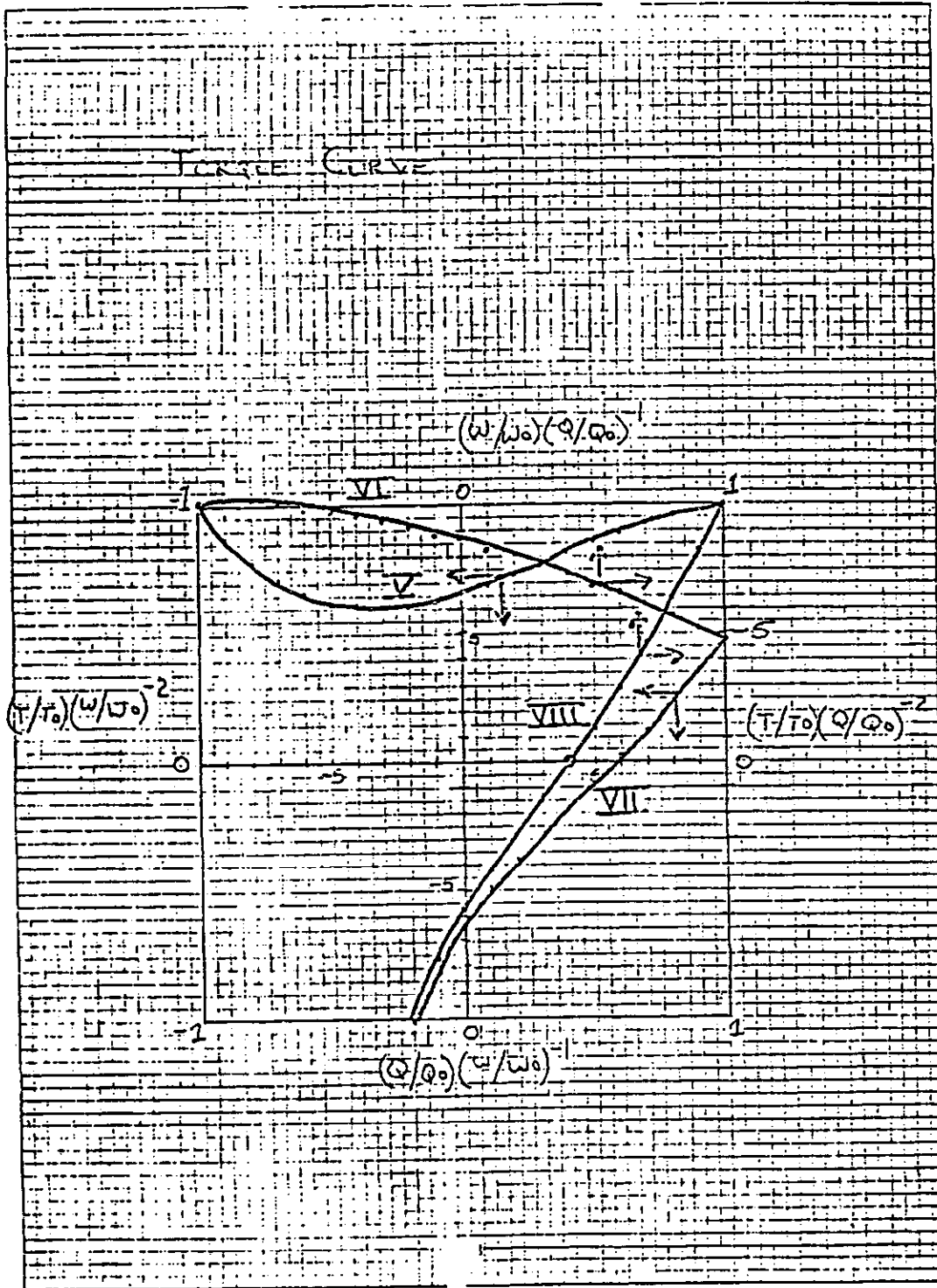**Figure 7.6** Head characteristics for a typical CANDU pump.
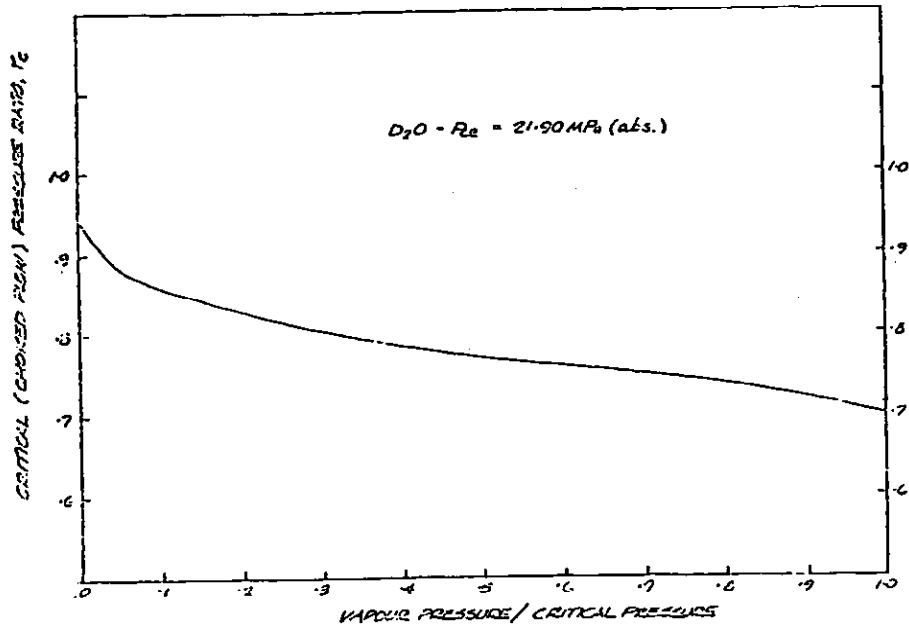
Figure 7.7 Torque characteristics for a typical CANDU pump.

Figure 7.8 Choked flow characteristics for a valve.
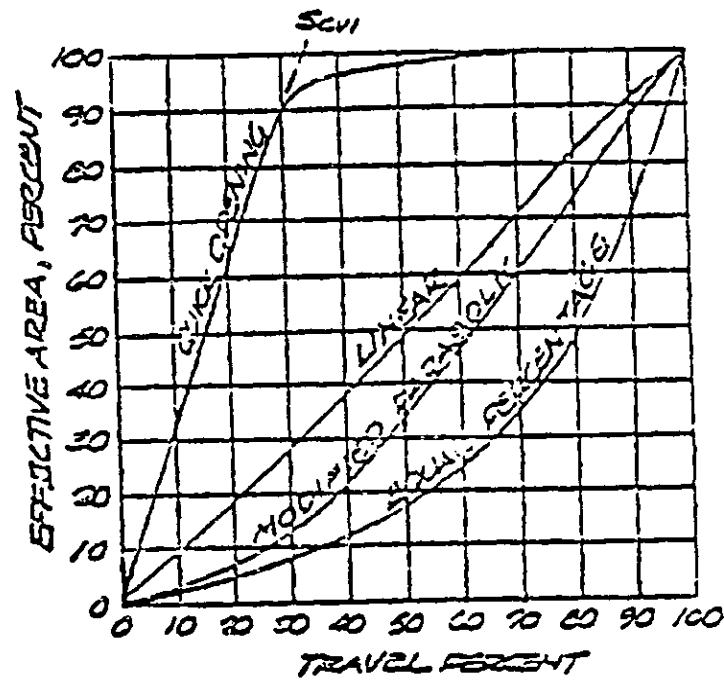
Figure 7.9 Control valve characteristics.
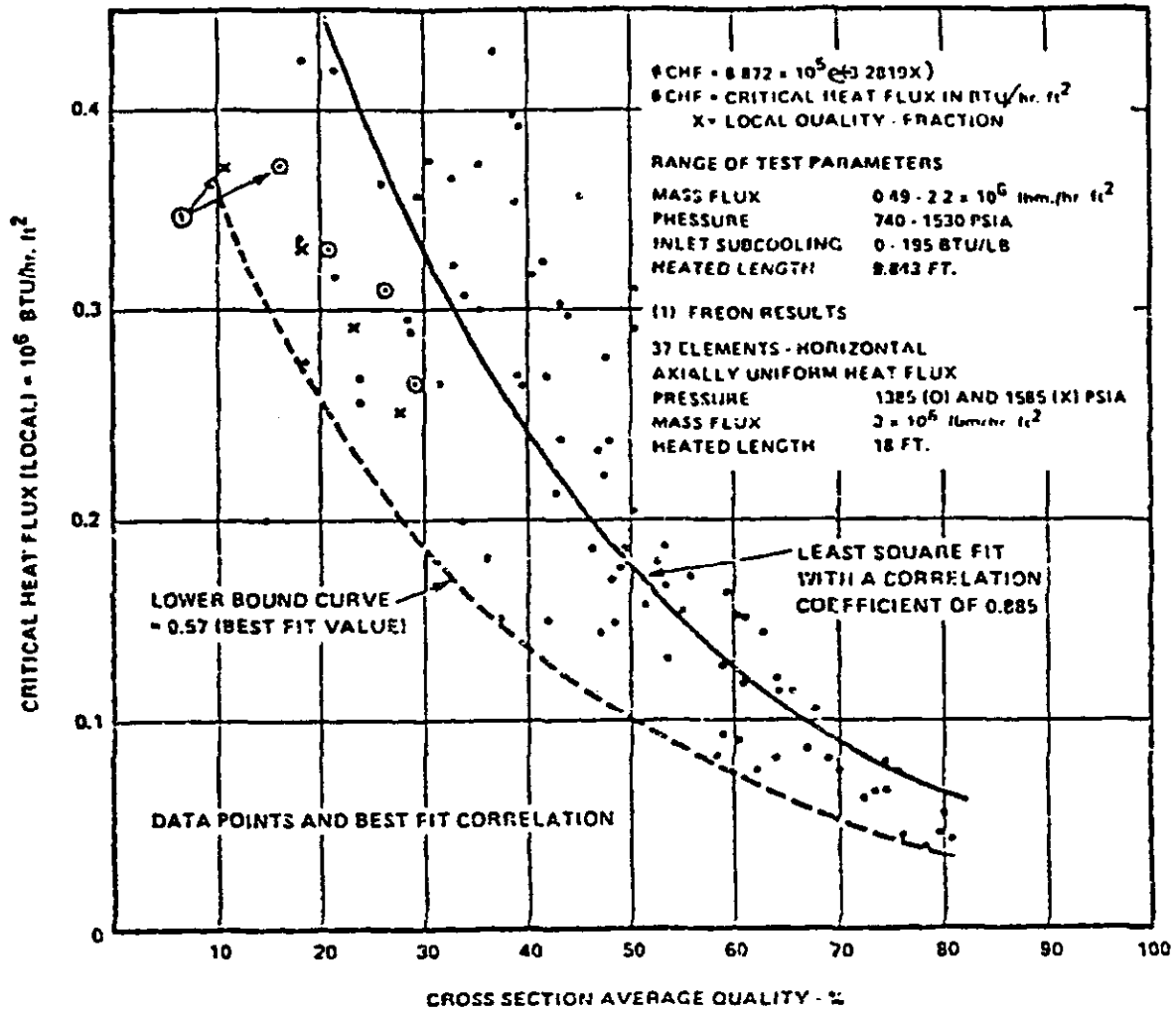
Figure 7.10 Critical heat flux.
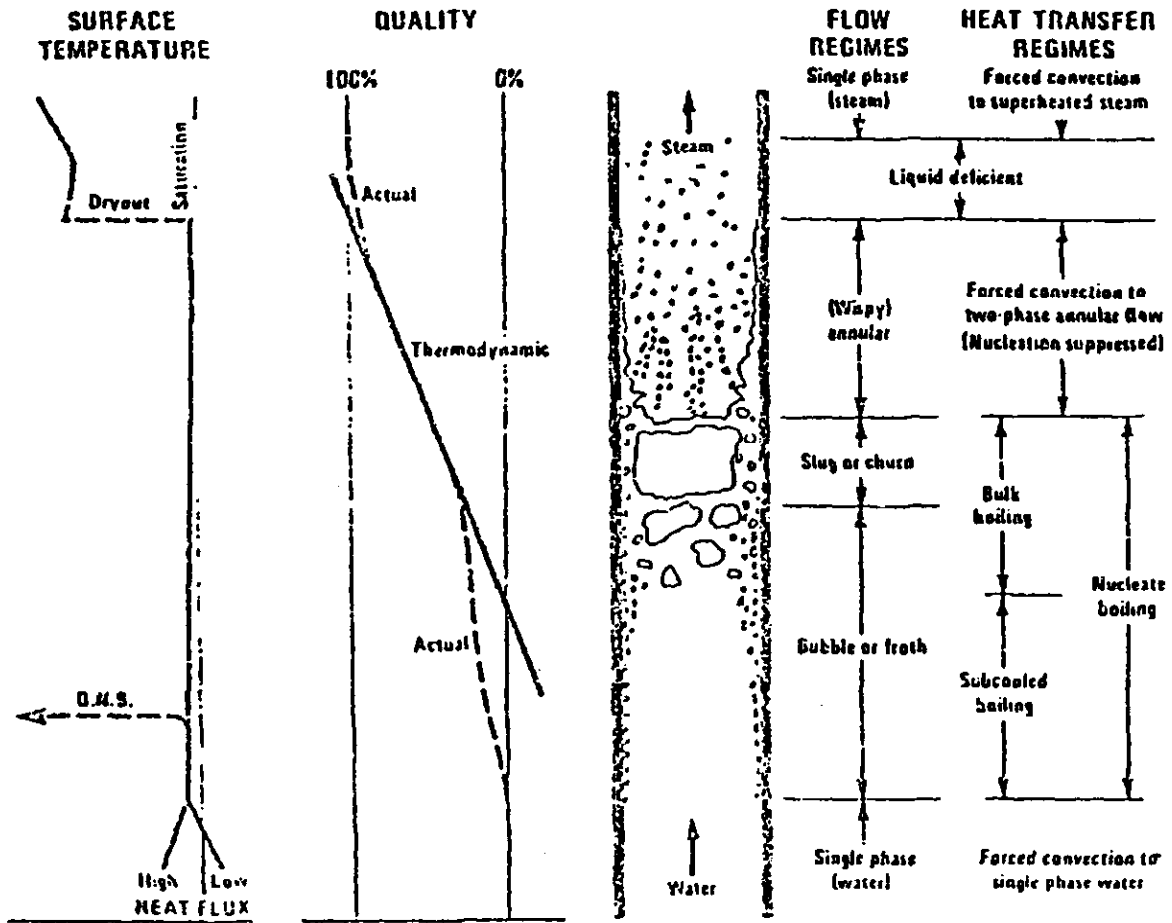
Figure 7.11 Possible thermalhydraulic regimes in a coolant channel.
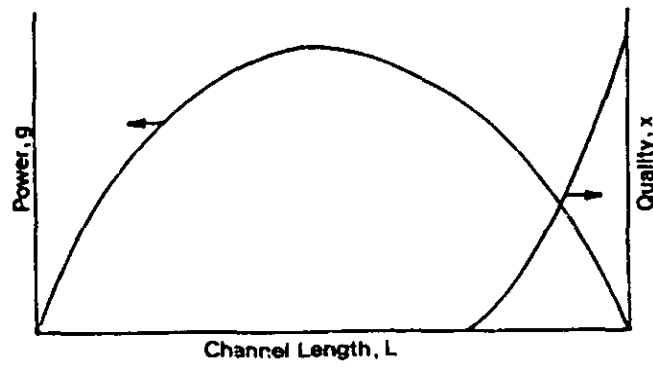
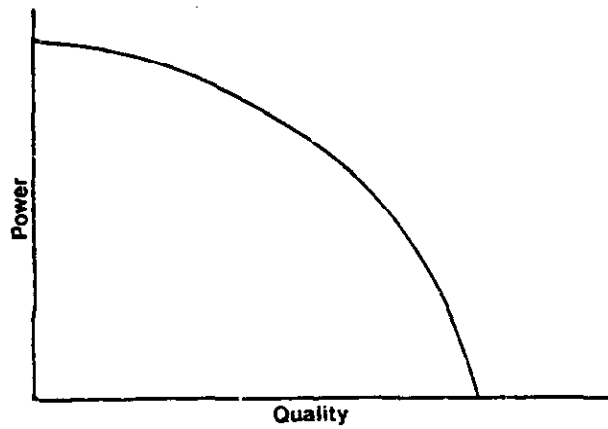**Figure 7.12** Power and quality versus length along a fuel channel.
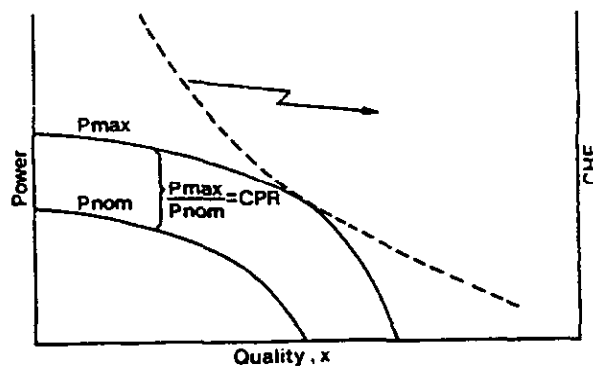

Figure 7.13 Power versus quality.

Figure 7.14 Critical Power Ratio determination.