Chapter 4 Thermodynamics

4.1 Introduction

This chapter focusses on the turbine cycle:thermodynamics and heat engines. The objective is to provide enough understanding of the turbine cycle to enable an appreciation of the role that it plays in overall plant design and performance. To set the scene, some thermodynamic fundamentals are reviewed in the next few sections. Then heat engine cycles are discussed. Sections 4.2 to 4.7 are based on [SEA75].

4.2 Work

The infinitesimal amount of work done by a system (shown in figure 4.1) is:

$$\begin{aligned} dW &= f\,dx \\ &= P\,A\,dx \\ &= P\,d\forall \end{aligned} \tag{4.1}$$

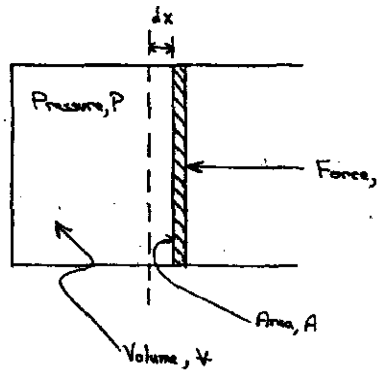

Figure 4.1 A simple systems for doing work

In figure 4.2, the shaded area represents the work done in moving from state a to state b, i.e.

$$W_{ab} = \int_{\forall_a}^{\forall_b} P\,d\forall \tag{4.2}$$

In the MKS system of units:

Pressure, P [=] N/m^2 (N = newton)
Volume, $\forall$ [=] m^3
∴ Work, W [=] N - m = J (J = joule).

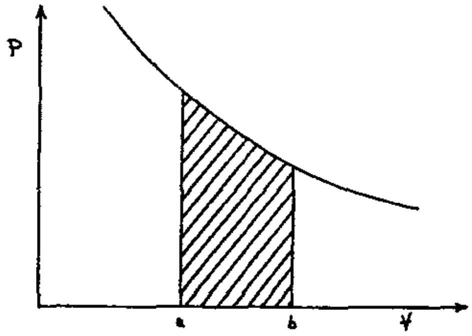

Figure 4.2 P∀ diagram

4.3 First Law of Thermodynamics

"The total work is the same in all adiabatic processes between any two equilibrium states having the same kinetic and potential energies". We generalize this to include the internal energy. This is just conservation of energy:

$$\begin{aligned} dE &= dQ - dW \\ \textit{where } E &= U + E_k + E_p \\ &= \textit{internal} + \textit{kinetic} + \textit{potential energies.} \end{aligned} \tag{4.3}$$

4.4 Enthalpy, h

Consider a substance undergoing a phase change (state 1 → state 2) at constant temperature. Since some change in volume generally occurs, the total change resulting from the energy input is

$$\int_1^2 dQ = \int_1^2 dU + \int_1^2 Pdv \tag{4.4}$$

On a unit mass basis, $q=Q/M$, $u = U/M$ and $v = \forall/M$, where M = mass. Thus:

$$q_2 - q_1 = u_2 - u_1 + Pv_2 - Pv_1 \tag{4.5}$$

(assuming that P = constant for this process).

The combination $u + Pv$ occurs frequently and is called the specific enthalpy, h:

$$h \equiv u + Pv. \tag{4.6}$$

<u>Example</u>: Consider the change in phase from liquid water to water vapour at 100°C. The latent heat of vaporization is $22.6\text{x}10^5$J/kg. The vapour pressure at 100°C is 1 atm = $1.01\text{x}10^5\text{N/m}^2$ and $v_g = 1.8$ m^3/kg and $v_f = 10^{-3}$/kg.

Thus the work done (in pushing back the atmosphere to make room for the vapour) is:

$$w = P(v_g - v_f) = 1.7\times105\ J/kg \tag{4.7}$$

Thus $u_g\text{-}u_f = 22.6\text{x}10^5 - 1.7\text{x}10^5$ J/kg
$= 20.9\ \text{x}10^5$ J/kg.

Thus 92% of the energy of transformation is used in increasing the internal energy and 8% is used in Pv work.

4.5 Energy Equation

The law of conservation of energy states:

Total energy of an isolated system is constant.

Consider the case, as shown in figure 4.3, where a mass m is added at 1 and leaves at 2. The system is at steady state. Thus:

$$\begin{aligned} \Delta E = 0 &= \sum \textit{energy inflow} - \sum \textit{energy outflow} \\ &= \frac{1}{2}mV_1^2 + mu_1 + Q + P_1V_1 + mgZ_1 \\ &\quad -\frac{1}{2}mV_2^2 - mu_2 + W - P_2V_2 - mgZ_2 \end{aligned} \tag{4.8}$$

On a per unit mass basis, where q = Q/m, w = W/m, we find:

$$\begin{gathered} (u_2 + Pv_2 + \frac{1}{2}V_2^2 + gZ_2) - (u_1 + Pv_1 + \frac{1}{2}V_1^2 + gZ_1) = q - w \\ or \\ (h_2 + \frac{1}{2}V_2^2 + gZ_2) - (h_1 + \frac{1}{2}V_1^2 + gZ_1) = q - w. \end{gathered} \tag{4.9}$$

<u>Special case:</u> turbine: $q \approx 0$, $Z_1 \approx Z_2$

$$w = (h_1 - h_2) + \frac{1}{2}(V_1^2 - V_2^2) \tag{4.10}$$

<u>Special case:</u> flow through a nozzle: $q = 0 = w$

$$V_2^2 = V_1^2 + 2(h_1 - h_2) \tag{4.11}$$

<u>Special case:</u> Bernoulli equation: $q = 0 = w$

$$\begin{gathered} h_1 + \frac{1}{2}V_1^2 + gZ_1 = h_2 + \frac{1}{2}V_2^2 + gZ_2 = constant \\ or \\ u + Pv + \frac{1}{2}V_2 + gZ = constant. \end{gathered} \tag{4.12}$$

If there is no friction, u = constant. In addition, if the fluid is incompressible:

$$Pv + \frac{1}{2}V^2 + gZ = constant$$

i.e. (4.13)

$$P + \frac{1}{2}\rho V^2 + \rho gZ = constant$$

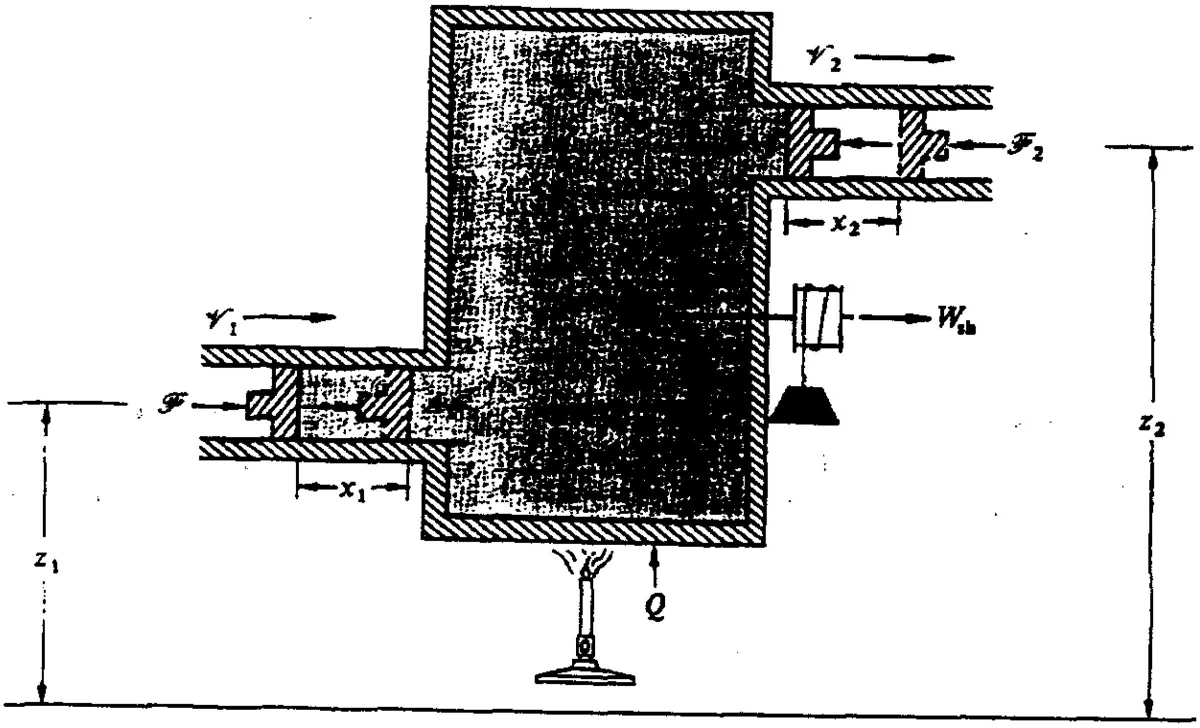

Figure 4.3 Steady flow process [Source: SEA75, figure 3-13]

4.6 The Carnot Cycle

The Carnot Cycle is illustrated in figure 4.4 and 4.5. This is the basis for all heat engine cycles and the turbine cycle in particular. The segments in the cycle are:

a-b	energy addition, $\Delta E = Q_2$
b-c	shaft work, $\Delta E - W_2$
c-d	condensation, $\Delta E = -Q_1$
d-a	pressurization, $\Delta E = 2W_1$.

The overall efficiency, η is:

$$\eta \equiv \frac{\textit{net work done}}{\textit{heat input}} = \frac{W}{Q_2} = \frac{Q_2 - Q_1}{Q_2} \tag{4.14}$$

($W = Q_2 - Q_1$ since there is no net change in energy in a complete cycle, ie.
$\Delta E = 0 = Q - W$ = net heat addition - net work done i.e. $Q = Q_2 - Q_1 = W$)

Q_1 is the reject heat.

It can be shown that:

$$\frac{|Q_2|}{|Q_1|} = \frac{T_2}{T_1} \tag{4.15}$$

Thus:

$$\eta = \frac{T_2 - T_1}{T_2} = 1 - \frac{T_1}{T_2} \tag{4.16}$$

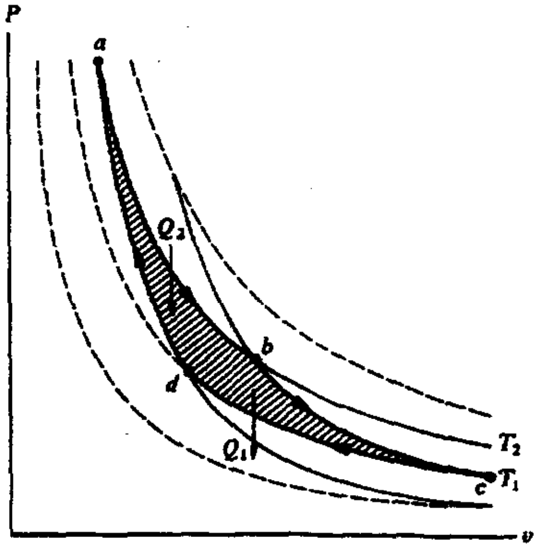

Figure 4.5 The Carnot cycle [Source SEA75, figure 4.6]

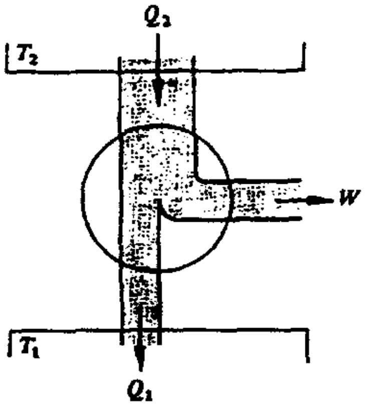

Figure 4.4 Schematic flow diagram of a heat engine [Source; SEA75, figure 4-7]

4.7 Entropy

Since Q_2 is heat flow <u>in</u> and Q_1 = heat flow <u>out</u>, their signs are opposite. Thus, from equation 4.15:

$$\frac{T_2}{T_1} = -\frac{Q_2}{Q_1} \rightarrow \frac{Q_1}{T_1} + \frac{Q_2}{T_2} = 0 \tag{4.17}$$

This can be generalized by splitting a general reversible cycle as in figures 4.6 and 4.7. We split the cycle up into many small Carnot cycles. The common boundaries cancel. For each small cycle:

$$\frac{\Delta Q_1}{T_1} + \frac{\Delta Q_2}{T_2} = 0 \tag{4.18}$$

Summing <u>all</u> cycles:

$$\begin{gathered} \sum \frac{\Delta Q}{T} = 0 \\ \oint \frac{dQ}{T} = 0 \end{gathered} \tag{4.19}$$

Since the closed integral = 0, dQ/T must be an exact differential and must be a <u>state variable</u>, i.e. a property of the state of the material, like u, P, T, ρ, etc. We define this to be the entropy, S. Thus:

$$\oint dS = 0 \tag{4.20}$$

So since S is a property of a system, we can express any equilibrium state in terms of S plus one other state variable (T,P or whatever).

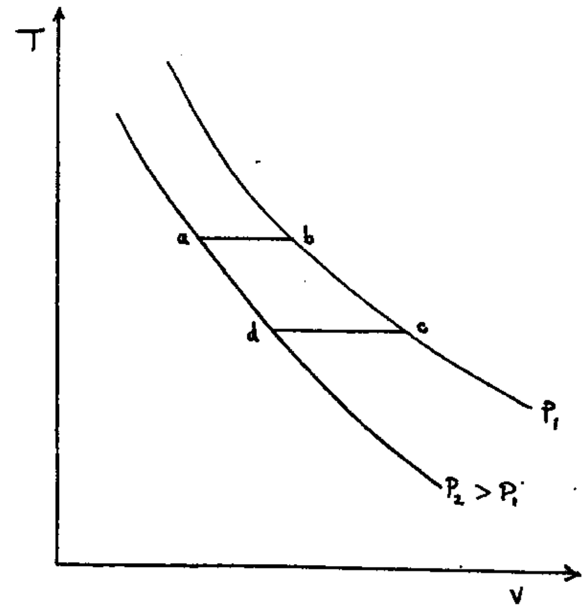

Figure 4.7 T-v diagram for the Carnot cycle

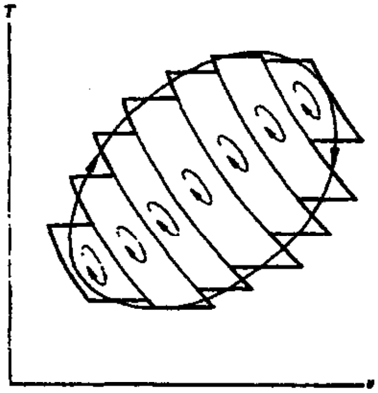

Figure 4.6 Any arbitrary reversible cyclic process can be approximated by a number of small Carnot cycles. [Source: SEA75, figure 5-3]

The Carnot cycle now becomes as shown in figure 4.8.

The T-S diagram gives the heat flow directly since:

$$\int_a^b TdS = \int_a^b dQ = Q_2$$

$$\text{and} \quad \int_c^d TdS = Q_1$$

$$\therefore \quad \oint TdS = \int_a^b + \int_b^c = \int_c^d + \int_d^a$$
$$= Q_2 + 0 - Q_1 + 0 = Q_2 - Q_1$$
$$= \textit{net heat flow into the system.} \tag{4.21}$$

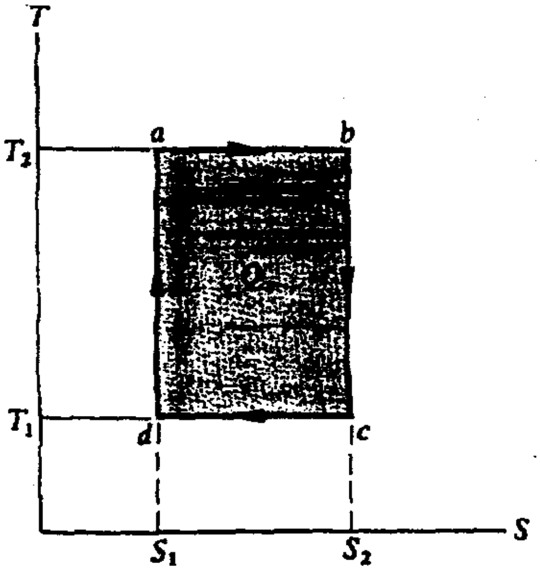

Figure 4.8 The temperature-entropy diagram for the Carnot cycle [SEA75, figure 5-4]

Note this is consistent with the previous definition of η:

$$\eta = \frac{Q_2 - Q_1}{Q_2} = \frac{T_2 (S_2 - S_1) - T_1(S_2 - S_1)}{T_2(S_2 - S_1)}$$
$$= \frac{T_2 - T_1}{T_2} = 1 - \frac{T_1}{T_2} \tag{4.22}$$

4.8 Reactor Power Cycle

[for the remainder of this chapter, the reference source is RUS79]

The thermodynamic power cycle in reactor systems is similar to the Carnot cycle. As sketched in figure 4.9, the steam generator boils the working fluid (water) isothermally (sort of), the turbine expands the fluid automatically and performs shaft work, the condenser extracts the reject heat and condenses the fluid, and the water pump returns the fluid to the steam generator at pressure. Of course, the cycle is not reversible but the principles of the cycle are the same. The typical cycle used in power plants is called the Rankine Cycle. The T-s and h-s diagrams for an ideal simple Rankine cycle is given in figure 4.10.

The h-s diagram is useful for calculation purposes while the T-s diagram is useful for illustration purposes.

In the ideal Rankine cycle, saturated steam (shown as point 1) enters the turbine and expands isentropically to position 2_s. At point 2_s, the wet steam enters the condenser where heat is removed until the fluid is condensed to a saturated liquid at point 3. After leaving the condenser, the fluid is condensed isentropically from pressure P_2 to the boiler pressure P_1. The high pressure liquid at point 4_s enters the boiler where the fluid is vaporized and emerges as steam at point 1.

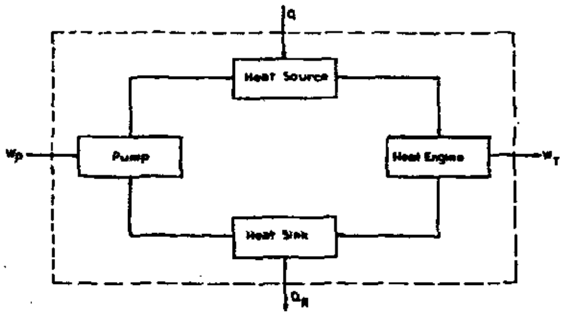

Figure 4.9 Schmatic diagram for a reactor power cycle [Source: RUS79, figure 2.7]

The shaded area represents the net work done ($W = Q_2 - Q_1$), the total area under the cycle curve represents the heat addition, Q_2, and the unshaded area is the rejected heat, Q_1.

From the h-s diagram it is straightforward to determine (on a unit mass basis):

$$\begin{aligned} \textit{Shaft work of the turbine} &= W_T = h_1 - h_{2s} \\ \textit{pumping work} &= W_p = h_{4s} - h_3 \\ \textit{heat input} &= Q\text{-}2 = h_1 - h_{4s} \\ \eta = \frac{W_T - W_p}{Q_2} &= \frac{W_{NET}}{Q_2} = \frac{(h_1 - h_{4s}) - (h_{4s}) - h_3)}{(h_1 - h_{4s})} \end{aligned} \tag{4.23}$$

Note: The above expression for η can be arranged to give:

as expected.

$$\eta = \frac{(h_1 - h_{4s}) - (h_{2s} - h_3)}{h_1 - h_{4s}} = \frac{Q_2 - Q_1}{Q_2} \tag{4.24}$$

Turbine performance is frequently given as the turbine heat rate:

$$\begin{aligned} \textit{Turbine heat rate} &\equiv \frac{\textit{heat supplied to boiler}}{\textit{net work out}} \\ &= \frac{1}{\eta} \end{aligned} \tag{4.25}$$

Power cycle performance is improved in practice by

(1) raising the boiler pressure
(2) Lowering the exhaust pressure
(3) using superheat
(4) using reheat

(1), (3) & (4) effectively raise the inlet temperature while (2) effectively lowers the outlet temperature with attendant effect on the cycle efficiency.

The condenser pressure is limited by the temperature of available cooling water, size and cost of the condenser, and size of the vacuum pumps required to deaerate the condenser. Consequently, the practical lower limit in condenser pressure is a few centimetres of Hg abs. Consequently, (1), (3) & (4) are used in achieve increases in efficiency.

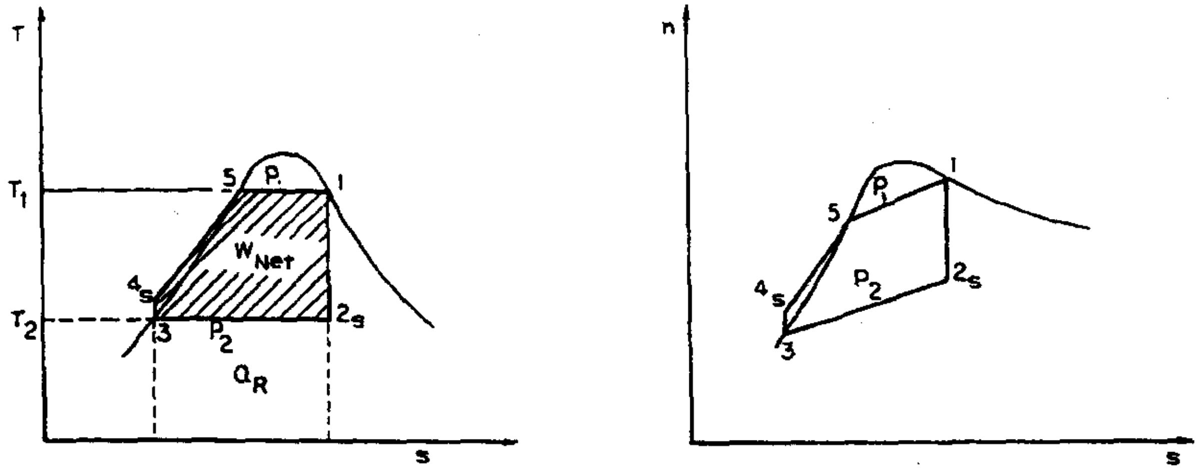

Figure 4.10 T-s and h-s diagram representations for the ideal Rankin cycles. Note: We are assuming fluid velocities are zero; i.e., the diagram illustrates stagnation properties. [Source: RUS79, figure 2.11]

4.9 Raising Boiler Pressure

The effect of increasing the boiler pressure on Rankine cycle efficiency is readily shown on the T-s diagram (see figure 4.11). Increasing boiler pressure results in an increase in net work (represented by the shaded area) with a corresponding decrease in heat rejected.

However, for the indirect power cycle, the downside of raising the boiler pressure (and temperature since the steam is saturated) is that it forces the primary side temperature up to provide sufficient ΔT to transfer the heat from the primary to secondary side. This higher primary side temperature push the fuel closer to its limits and increases the tendency for the fluid to boil. To counter this, if necessary, the primary side pressure would have to be increased and pressure vessel walls would have to be thicker. In a pressure vessel type reactor, this can be costly or lead to reduced reliability. In pressure tube reactors, the main drawback is the increased parasitic neutron absorption and consequent higher burnup.

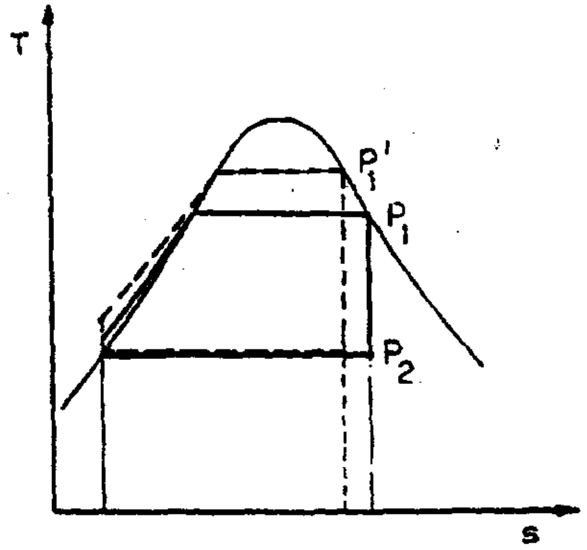

Figure 4.11 Effects of increasing boiler pressure on the Rankin cycle [Source: RUS79, figure 2.13]

4.10 Superheat

Figure 4.12 illustrates the Rankine cycle with superheat. Superheat causes a net increase in temperature at which heat is being received with a resulting improvement in cycle efficiency. Another important factor is that the amount of moisture in the fluid leaving the turbine is reduced which increases turbine efficiency and reduces erosion. However, in order to make use of superheat, one must have a high temperature heat source or reduce boiler pressure.

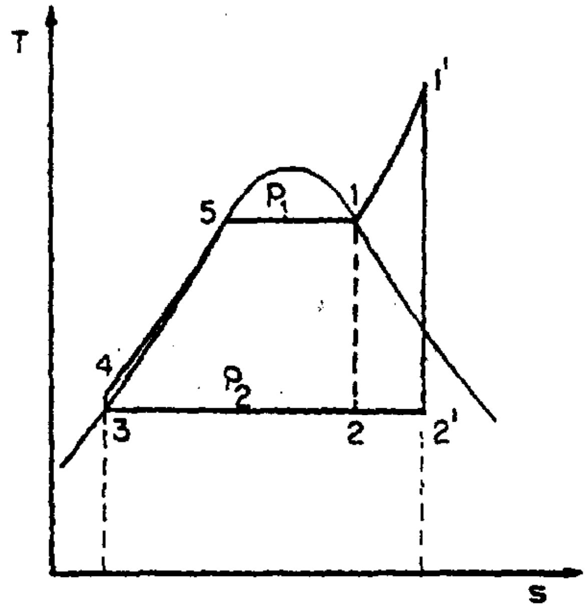

Figure 4.12 Rankin cycle with superheat [Source: RUS79, figure 2.14]

4.11 Reheat

The effective temperature of heat addition is increased and the moisture content further reduced by using reheat in the Rankine cycle. A schematic diagram of the power plant and appropriate temperature-entropy diagram is shown in figure 4.13 . High pressure, superheated steam is expanded in a high-pressure turbine to an intermediate pressure p'_2 and the fluid then returned to a second stage boiler and superheat and reheated to state 1". The reheated steam is then expanded in a low-pressure turbine to the final exhaust pressure p''_2. The moisture content of the working fluid is drastically reduced by use of reheat and this approach is used in all fossil-fuelled and many nuclear power plants.

The approach used to compute the work and efficiency of reheat cycles is the same as used in the example problem for the simple Rankine cycle. One calculates the work produced in each turbine separately and the required pumping work. Heat is added to the fluid at two different stages of the cycle and is given by the difference in enthalpy between states 1' and 4 and states 1" and 2'.

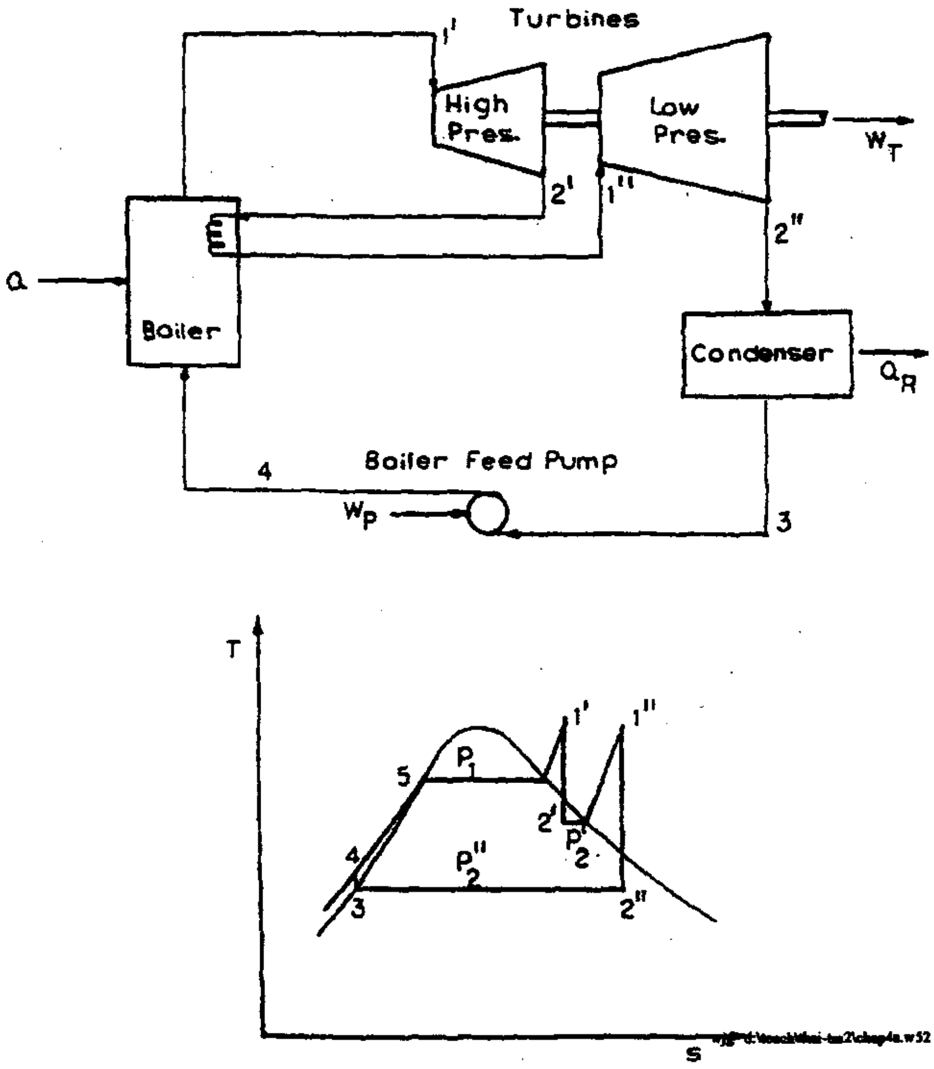

Figure 4.13 Rankin cycle with reheat [Source: RUS79, figure 2.15]

4.12 Regeneration

Modifications to the cycle can also be made to reduce cycle irreversibility. One of the principle sources is the sensible heat addition required to bring the boiler feedwater up to saturation temperature. This is accomplished by using some of the flow through the turbine to heat the feedwater. To achieve reversibility, the setup would be as in figure 4.14 but this is impractical. A practical setup is shown in figure 4.15. Analysis is beyond the scope of this course.

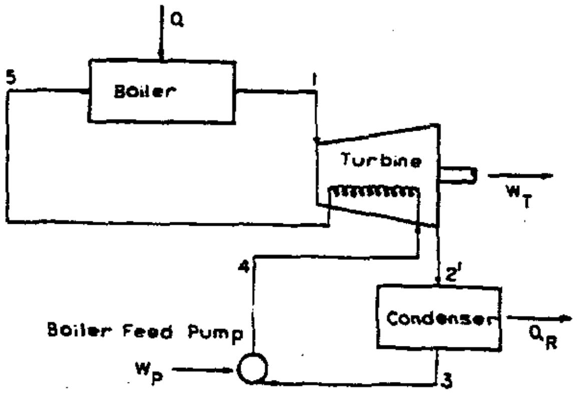

Figure 4.14 Schematic diagram of a power plant with ideal regeneration [Source: RUS79, figure 2.17]

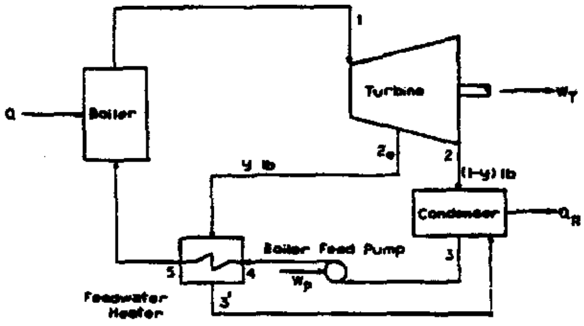

Figure 4.15 Single heater regenerative cycle [Source: RUS79, figure 2.18]