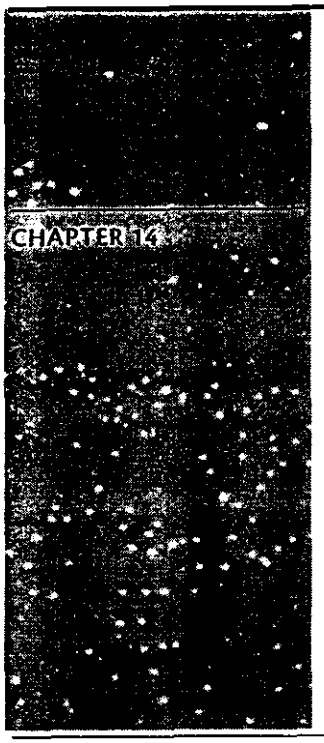

Modal Synthesis

THE IDEA BEHIND MODAL SYNTHESIS IS to construct the neutron flux from predetermined static shapes, depending on space only. The coefficients multiplying these shapes, the modal amplitudes, will depend on time. Thus it is left to find the differential equations which drive the modal amplitudes, and also to solve these equations.

Modal Notation

In a very general approach, modal synthesis is written as

$$[\phi] = \sum_{k=1}^{K} [\Psi_k(\vec{r})][T_k(t)] \qquad \text{(EQ 91)}$$

where $[\Psi_k(\vec{r})]$ is a G × G diagonal matrix containing the elements of the k^{th} mode, and $[T_k(t)]$ is a column vector of G elements, the modal amplitude vector.

There is in the expansion (91) K shape functions or modes, which are time independent. Consequently, the modal expansion can only be an approximate relationship, which cannot be valid at all spatial points at all times. We therefore demand that the modal expansion be correct in an integral sense rather tan in a local sense.

To be more precise, equation (91) is substituted in the time dependent diffusion equations, and equality is required after it has been spatially integrated over the entire reactor volume. The equality between the modal expansion and the flux will not be attained, and it may very well be that there is not a single point in space where it is valid. The approximate relationship will still be a solution of the spatially integrated equations.

Mathematical Derivation

In order to determine the differential equations that govern the modal amplitudes, we start by introducing, for more generality, the K weight functions

$$[W_r(\vec{r})], r=1,\ldots, K$$

which are diagonal $G \times G$ matrices. We substitute (91) in the time dependent diffusion equations in matrix form, which we pre-multiply by $[W_r(\vec{r})]$ and that we integrate over space to get

$$\left(\sum_{k=1}^{K} \langle [W_r(\vec{r})][v]^{-1} \frac{\partial}{\partial t} [\Psi_k(\vec{r})][T_k(t)] \rangle \right)$$

$$= \sum_{k=1}^{K} \langle [W_r(\vec{r})] \left(\begin{array}{l} \nabla \cdot [D] \nabla [\Psi_k(\vec{r})][T_k(t)] - [\Sigma][\Psi_k(\vec{r})][T_k(t)] \\ \quad + (1-\beta)[\chi^p][\nu\Sigma_f]^T [\Psi_k(\vec{r})][T_k(t)] \end{array} \right) \rangle$$

$$+ \sum_{i=1}^{D} \lambda_i \langle [W_r(\vec{r})][\chi_i^d] C_i \rangle$$

and

$$\langle [W_r(\vec{r})][\chi_i^d] \frac{\partial}{\partial t} C_i \rangle = \sum_{i=1}^{K} \beta_i \langle [W_r(\vec{r})][\chi_i^d][\nu\Sigma_f]^T [\Psi_k][T_k] \rangle$$

$$- \lambda_i \langle [W_r(\vec{r})][\chi_i^d] C_i \rangle$$

Let us define the matrix

$$[\mathcal{A}_{rk}] = \langle [W_r][v]^{-1} [\Psi_k] \rangle^{-1} \qquad \text{(EQ 92)}$$

We take the $[T_k(t)]$ out of the spatial integrals, and we pre-multiply the two equation system by the matrix $[\mathcal{A}_{rr}]$ to get, after having added and substracted the terms in β_i to the flux equation,

$$\sum_{k=1}^{K} [\mathcal{A}_{rr}][\mathcal{A}_{rk}]\frac{d}{dt}[T_k]$$

$$= \sum_{k=1}^{K} [\mathcal{A}_{rr}]\langle [W_k]\left\{ \begin{array}{c} \nabla \cdot [D]\vec{\nabla}[\Psi_k] - [\Sigma][\Psi_k] \\ + \left((1-\beta)[\chi^p] + \sum_{i=1}^{D} \beta_i [\chi_i^d] \times \right) \\ \times [\nu\Sigma_f]^T[\Psi_k] \end{array} \right\} \rangle [T_k]$$

$$- \sum_{k=1}^{K}\sum_{i=1}^{D} \beta_i [\mathcal{A}_{rr}]\langle [W_r][\chi_i^d][\nu\Sigma_f][\Psi_k]\rangle [T_k]$$

$$+ \sum_{i=1}^{D} \lambda_i [\mathcal{A}_{rr}]\langle [W_r][\chi_i^d][\Psi_k]C_i\rangle$$

and

$$\frac{d}{dt}[\mathcal{A}_{rr}]\langle [W_r][\chi_i^d][\Psi_k]C_i\rangle =$$

$$\beta_i \sum_{k=1}^{K} [\mathcal{A}_{rr}]\langle [W_r][\chi_i^d][\nu\Sigma_f]^T[\Psi_k]\rangle [T_k]$$

$$-\lambda_i [\mathcal{A}_{rr}]\langle [W_r][\chi_i^d]C_i\rangle$$

Before proceeding any further, we notice that point kinetics could well have been done without the average life of the neutrons, the Λ term. To see this, consider the point kinetics equations

$$\frac{d}{dt}T = \left(\frac{\rho}{\Lambda} - \frac{\beta}{\Lambda}\right)T + \sum_{i=1}^{D} \lambda_i C_i$$

$$\frac{d}{dt}C_i = \frac{\beta_i}{\Lambda}T - \lambda_i C_i$$

It is clear that only the ratios $\frac{\rho}{\Lambda}$ and $\frac{\beta_i}{\Lambda}$ are required in the theory. Furthermore, if we examine these ratios together with the point kinetics parameter definitions, we can see that these ratios do not involve the denominator

$$\left\langle \left((1-\beta)[\chi^p] + \sum_{i=1}^{D} \beta_i [\chi_i^d] \right) [\nu\Sigma_f]^T [S] \right\rangle$$

We thus take as example the point kinetics derivation, and we define the following quantities:

- a modal precursor concentration

$$[C_i^f] = [\mathcal{A}_{rr}]\langle [W_r][\chi_i^d] C_i \rangle$$

- a modal delayed neutron fraction

$$[\Lambda^{-1}\beta_i^{rk}] = \beta_i [\mathcal{A}_{rr}]\langle [W_r][\chi_i^d][\nu\Sigma_f]^T [\Psi_k] \rangle$$

and

$$[\Lambda^{-1}\beta^{rk}] = \sum_{i=1}^{D} [\Lambda^{-1}\beta_i^{rk}]$$

- a modal reactivity matrix

$$[\Lambda^{-1}\rho^{rk}] = [\mathscr{A}_{rr}]\langle[W_k]\left\{\begin{array}{c}\nabla\cdot[D]\vec{\nabla}[\Psi_k] - [\Sigma][\Psi_k] \\ +\left((1-\beta)[\chi^p] + \sum_{i=1}^{D}\beta_i[\chi_i^d]\right) \\ \times[\nu\Sigma_f]^T[\Psi_k]\end{array}\right\}\rangle$$

With these definitions, we can rewrite the flux and delayed precursor equations in the form

$$\sum_{p=1}^{K}[\mathscr{A}_{rr}][\mathscr{A}_{rp}]\frac{d}{dt}[T_p] = \sum_{k=1}^{K}([\Lambda^{-1}\rho^{rk}] - [\Lambda^{-1}\beta^{rk}])[T_k] + \sum_{i=1}^{D}\lambda_i[C_i^r] \quad \text{(EQ 93)}$$

and

$$\frac{d}{dt}[C_i^r] = \sum_{k=1}^{K}[\Lambda^{-1}\beta_i^{rk}][T_k] - \sum_{i=1}^{D}\lambda_i[C_i^r] \quad \text{(EQ 94)}$$

Equation Structure

We have established in all generality the differential equations for the modal amplitudes of any of the modal synthesis methods. Note that no hypothesis other than the time independence of the modes has been necessary. Linear independence is required however, because in certain cases the inverses of the matrices that appear in the modal param-

eter definitions would become singular. Orthogonality of the modes is not required for the determination of the modal amplitudes, and of the resulting synthesis.

It is worth noting that the modal amplitudes obey a differential system of equations whose *structure* is very much the same as that of the point kinetics equations. This provides an indication that the solution techniques could be generalized to modal equations without too much difficulties.

In practice, the solution of these equations with a given mode set is done directly from the system of coupled differential equations resulting from the matrix decomposition element by element. The matrix formulation is useful because it makes for a compact notation, and because it emphasizes the resemblance with the point kinetics equations.

Also, and therein lies a great advantage for modal methods, all the modal parameters can be pre-calculated for a given reactor, once the set of modes has been chosen. For example, an absorber rod (like an adjuster rod in a CANDU reactor) moves in a very precise area of the core, and it is easy to calculate the integrals that appear in the modal reactivity. In an actual simulation, these modal reactivities only have to be interpolated in the pre-calculated tables, which saves a great deal of calculational effort.

Choice of Modes

Many types of modes can be used in modal analysis. In particular, pre-calculated flux shapes corresponding to the reactor perturbed in different ways could be used. This is known as "temporal synthesis", which is an excellent method mostly used in light water reactor simulations. It requires a set of modes that are "close" to the perturbations being studied, and there is a certain degree of experience required in choosing adequate modes. Also, it can be the case that such modes are almost linearly dependent, which makes the differential equations difficult to solve. A decontamination of the different modes may be required to minimize this problem...

Lambda Modes

A method very much in use for the analysis of CANDU reactors is synthesis by the so-called λ modes. These are the solutions of the static equations for the *non perturbed* reactor,

$$[L_0][\Psi_n] = \frac{1}{\lambda_n}[M_0][\Psi_n]$$

for the successive values of λ_n. These modes are the eigenvectors of the static equations, corresponding to eigenvalues of lower value than the K_{eff}. These eigenvectors take negative values in certain areas of the core. It is quite difficult to calculate more than about twenty of these modes, since the eigenvalues tend to become closer to one another: the

dominance ratio is pretty close to one, and convergence becomes very slow.

The adjoint modes are calculated at the same time, to be used as weight functions. These are the solutions of the problem

$$[L_0][\Psi_n^*] = \frac{1}{\lambda_n^*}[M_0][\Psi_n^*]$$

It is easy to show the orthogonality relationship,

$$\left(\frac{1}{\lambda_m^*} - \frac{1}{\lambda_n}\right)\langle[\Psi_m^*][M_0][\Psi_n]\rangle = 0$$

This tells us that when $m = n$, then $\lambda_m = \lambda_n$, or that the eigenvalues of the two systems are the same. On the other hand, if $m \neq n$, then $\langle[\Psi_m^*][M_0][\Psi_n]\rangle = 0$.

If we use these adjoint fluxes as weight functions, we will find that

$$\Lambda_{rk} = 0, r \neq k$$

and this simplifies significantly the differential equations for the modal amplitudes, particularly on the right hand side, which now only has diagonal elements.