CHAPTER 5

Matrix Form of the Equations

In order to facilitate the mathematical formulation of the methods that we will examine in this course, it is very useful to express the space-time kinetics equations in matrix form. This makes them look independent of any particular energy group, and produces a much more compact notation. To do this we have to introduce the following vectors and matrices:

- A diagonal matrix of the velocities

$$[v] = \begin{bmatrix} v_1 & & & \\ & v_2 & & \\ & & \cdot & \\ & & & v_G \end{bmatrix}$$

- A column vector of the fluxes,

$$[\phi] = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \cdot \\ \phi_G \end{bmatrix}$$

- A column vector of the net currents,

$$[\vec{J}] = \begin{bmatrix} \vec{J}_1 \\ \vec{J}_2 \\ \cdot \\ \vec{J}_G \end{bmatrix}$$

- A square matrix of the cross-sections,

$$[\Sigma] = \begin{bmatrix} \Sigma_{t1} - \Sigma_{s1 \leftarrow 1} & -\Sigma_{s1 \leftarrow 2} & \cdots & -\Sigma_{s1 \leftarrow G} \\ -\Sigma_{s2 \leftarrow 1} & \Sigma_{t2} - \Sigma_{s2 \leftarrow 2} & \cdots & -\Sigma_{s2 \leftarrow G} \\ \cdot & \cdot & \cdot & \cdot \\ -\Sigma_{sG \leftarrow 1} & -\Sigma_{sG \leftarrow 2} & \cdots & \Sigma_{tG} - \Sigma_{sG \leftarrow G} \end{bmatrix} \quad \text{(EQ 19)}$$

- A column vector of the prompt energy spectrum,

$$[\chi^p] = \begin{bmatrix} \chi_1^p \\ \chi_2^p \\ \cdot \\ \chi_G^p \end{bmatrix}$$

- A column vector of ν times the fission cross-section,

$$\left[\nu\Sigma_f\right] = \begin{bmatrix} \nu\Sigma_{f1} \\ \nu\Sigma_{f2} \\ \cdot \\ \nu\Sigma_{fG} \end{bmatrix}$$

- D column vectors of the delayed neutron spectra,

$$\left[\chi_i^d\right] = \begin{bmatrix} \chi_{i1}^d \\ \chi_{i2}^d \\ \cdot \\ \chi_{iG}^d \end{bmatrix}$$

- A diagonal matrix of the diffusion coefficients,

$$\left[D\right] = \begin{bmatrix} D_1 & & & \\ & D_2 & & \\ & & \cdot & \\ & & & D_G \end{bmatrix}$$

With these definitions, it is very easy to re-write the space-time kinetics equations of the preceding chapter in a much more compact form,

$$
\begin{aligned}
[v]^{-1}\frac{\partial}{\partial t}[\phi] &= \nabla \cdot [D]\vec{\nabla}[\phi] - [\Sigma][\phi] + (1-\beta)[\chi^p][\nu\Sigma_f]^T[\phi] \\
&\quad + \sum_{i=1}^{D} [\chi_i^d]\lambda_i C_i \\
[\vec{J}\,] &= -[D]\vec{\nabla}[\phi] \\
\frac{\partial}{\partial t}C_i &= \beta_i[\nu\Sigma_f]^T[\phi] - \lambda_i C_i
\end{aligned}
\qquad \text{(EQ 20)}
$$