CHAPTER 4

Energy Condensation

The space-time kinetics equations (1), (2) and (3) that we have obtained have been formulated with a continuous energy variable. However, it is not practical to solve these equations. In order to further simplify the equations, it is useful to perform a discretisation of the energy variable. This process is examined in this chapter.

Energy Partitioning

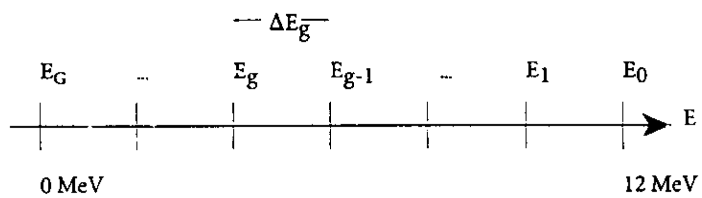
We start the partitioning process by separating the energy domain into a number G widths, which we call energy groups, as illustrated on Figure 1, page 39. The choice of the energy group boundaries is left to the analysts, and are usually chosen by taking into account the variations of the cross-sections of the important isotopes that are included in the reactor model. The presence of fissile isotope resonances will play a determinant role in this process. We also note that the microscopic cross-section are measured by many of the nuclear physics laboratories in the world, and are listed in the ENDF-B VI data files. These microscopic cross-sections are continuous in energy. A preliminary treatment with hyperfine group widths is done with the computer code NJOY to generate specialized cross-section libraries for lattice cell calculations, which will have about one hundred energy groups.

Lattice codes such as DRAGON, WIMS, APOLLO, CASMO, use these specialized libraries to perform transport theory calculations, and will also perform another energy condensation to generate two group cross-sections and diffusion coefficients that are routinely used in reactor calculations.

The process that we will describe here is relatively simplified by comparison to methods currently used in these codes. For example, the use of flux shapes within the energy groups is not covered here. Furthermore, the energy condensation done at the cell code level uses static fluxes in the process, while we could believe that time dependent fluxes should be used to calculate the parameters. This turns out to be a very small correction that we will entirely neglect.

Note also the convention that high group numbers correspond to smaller energies. The rational behind this convention is that neutrons are born at high energies and are subsequently slowed down toward the lower energies.

FIGURE 1. Energy Domain Partition



Multigroup Reduction

The multigroup reduction process is to apply the operator

$$\int_{E_g}^{E_{g-1}} dE \tag{EQ 4}$$

to the space-time kinetics equations.

We also introduce the following definitions:

- The group flux

$$\phi_g(\vec{r}, t) \equiv \int_{E_g}^{E_{g-1}} \phi(\vec{r}, E, t)dE \tag{EQ 5}$$

  Let us note that the group flux is not an energy density anymore; it's dimensions are not the same as that of $\phi(\vec{r}, E, t)$, the continuous energy flux.

- The net current of group g

$$\vec{J}_g(\vec{r}, t) = \int_{E_g}^{E_{g-1}} \vec{J}(\vec{r}, E, t)dE \qquad \text{(EQ 6)}$$

This group current is also not a density anymore, just like the group flux, and it's dimensions are not the same as the continuous energy current.

- The total cross-section of group g :

$$\Sigma_{tg}(\vec{r}) \equiv \frac{\int_{E_g}^{E_{g-1}} \Sigma_t \phi(\vec{r}, E, t)dE}{\int_{E_g}^{E_{g-1}} \phi(\vec{r}, E, t)dE} = \frac{\int_{E_g}^{E_{g-1}} \Sigma_t \phi(\vec{r}, E, t)dE}{\phi_g} \qquad \text{(EQ 7)}$$

- The fission cross-section of group g :

$$\Sigma_{fg}(\vec{r}) \equiv \frac{\int_{E_g}^{E_{g-1}} \Sigma_f \phi(\vec{r}, E, t)dE}{\int_{E_g}^{E_{g-1}} \phi(\vec{r}, E, t)dE} = \frac{\int_{E_g}^{E_{g-1}} \Sigma_f \phi(\vec{r}, E, t)dE}{\phi_g} \qquad \text{(EQ 8)}$$

- The prompt neutron spectrum of group g :

$$\chi_g^p = \int_{E_g}^{E_{g-1}} \chi^p(E)dE \qquad \text{(EQ 9)}$$

The group spectrum is such that the sum over all energy groups gives 1, since the sum returns the complete integral.

- The delayed neutron spectrum of family i and of group g :

$$\chi_{gi}^d = \int_{E_g}^{E_{g-1}} \chi_i^d(E)dE \qquad \text{(EQ 10)}$$

The same comment applies to group delayed neutron spectra as for the prompt neutron spectrum.

- The diffusion coefficient of group g

$$\frac{1}{D_g(\vec{r})} = \frac{\int_{E_g}^{E_{g-1}} \frac{1}{D(E)}\phi(\vec{r}, E, t)dE}{\int_{E_g}^{E_{g-1}} \phi(\vec{r}, E, t)dE} = \frac{\int_{E_g}^{E_{g-1}} \frac{1}{D(E)}\phi(\vec{r}, E, t)dE}{\phi_g} \quad \text{(EQ 11)}$$

The inverse of the coefficient is used because this inverse is proportional to the transport cross-section.

- The scattering cross-section from group g' to group g. We start by defining the scattering cross-section for neutrons in group g' towards an energy between E and E + dE,

$$\Sigma_{sg'}(\vec{r}, E)dE \equiv \frac{\int_{E_{g'}}^{E_{g'-1}} \Sigma_s(E' \to E)\phi(\vec{r}, E')dE'}{\int_{E_{g'}}^{E_{g'-1}} \phi(\vec{r}, E')dE'} dE$$

We the integrate this over the energy group g of neutron arrival

$$\Sigma_{sg \leftarrow g'}(\vec{r}) \equiv \frac{\int_{E_g}^{E_{g-1}} \int_{E_{g'}}^{E_{g'-1}} \Sigma_s(E' \to E)\phi(E')dE'dE}{\phi_{g'}} \quad \text{(EQ 12)}$$

Diffusion Equations in Terms of Energy Groups

Space-time Kinetics Equations

Applying the integral operator (4) together with the definitions of the group parameters given by equations (55) to (12), the space-time kinetics equations become:

- for the fluxes

$$\frac{1}{v_g}\frac{\partial}{\partial t}\phi_g(\vec{r}, t) = -\nabla \cdot \vec{J}_g(\vec{r}, t) - \Sigma_{tg}\phi_g + \sum_{g'=1}^{G} \Sigma_{sg \leftarrow g'}\phi_{g'} + \chi_g^P(1-\beta)\sum_{g'=1}^{G} \nu\Sigma_{fg'}\phi_{g'} + \sum_{i=1}^{D} \chi_{ig}^D \lambda_i C_i(\vec{r}, t) \quad \text{(EQ 13)}$$

- for the delayed neutron precursor concentrations,

$$\frac{\partial}{\partial t}C_i(\vec{r}, t) = \beta_i \sum_{g'=1}^{G} \nu\Sigma_{fg'}\phi_{g'} - \lambda_i C_i(\vec{r}, t) \quad \text{(EQ 14)}$$

for the currents,

$$\vec{J}_g(\vec{r}, t) = -D_g\vec{\nabla}\phi_g \quad \text{(EQ 15)}$$

Static Equation

Starting from the space-time kinetics equations (13) and (14), it is possible to get the static equations, by setting to zero the time derivatives. The precursor equations become

$$0 = \beta_i \sum_{g'=1}^{G} \nu\Sigma_{fg'}\phi_{g'} - \lambda_i C_i(\vec{r}, t)$$

$$\Rightarrow C_i(\vec{r}, t) = \frac{\beta_i}{\lambda_i} \sum_{g'=1}^{G} \nu\Sigma_{fg'}\phi_{g'}$$

which we substitute in the flux equations

$$\begin{aligned} 0 = & -\nabla \cdot \vec{J}_g(\vec{r}, t) - \Sigma_{tg}\phi_g \\ & + \sum_{g'=1}^{G} \Sigma_{sg \leftarrow g'}\phi_{g'} + \chi_g^P(1-\beta) \sum_{g'=1}^{G} \nu\Sigma_{fg'}\phi_{g'} \\ & + \sum_{i=1}^{D} \chi_{ig}^d \lambda_i \frac{\beta_i}{\lambda_i} \sum_{g'=1}^{G} \nu\Sigma_{fg'}\phi_{g'} \end{aligned}$$

which becomes

$$\begin{aligned} & \nabla \cdot \vec{J}_g(\vec{r}, t) + \Sigma_{tg}\phi_g \\ = & \sum_{g'=1}^{G} \Sigma_{sg \leftarrow g'}\phi_{g'} + \left\{ \chi_g^P(1-\beta) + \sum_{i=1}^{D} \chi_{ig}^d \beta_i \right\} \sum_{g'=1}^{G} \nu\Sigma_{fg'}\phi_{g'} \end{aligned}$$

If we realize that the neutron spectrum for the static case must take into account the spectrum of all neutrons, whether prompt or delayed, we can write

$$\chi_g = \left\{\chi_g^P(1-\beta) + \sum_{i=1}^{D} \chi_{ig}^d \beta_i\right\} \tag{EQ 16}$$

and the static equations then become

$$\nabla \cdot \vec{J}_g(\vec{r}) + \Sigma_{tg}\phi_g = \sum_{g'=1}^{G} \Sigma_{sg \leftarrow g'}\phi_{g'} + \chi_g \sum_{g'=1}^{G} \nu\Sigma_{fg'}\phi_{g'} \tag{EQ 17}$$

$$\vec{J}_g(\vec{r}, t) = -D_g \vec{\nabla}\phi_g \tag{EQ 18}$$