Mathematics - Course 321

USE OF LOGARITHMIC SCALED GRAPH PAPER

We have discussed the use of graphs for many purposes in previous courses. In all the cases considered, the graphs have been plotted on squared paper on which all the divisions are equal. These divisions may be 1/4" long or 1/6", they may be 1/10" or 1 millimeter, but in all cases they are all equal divisions. The scales used on such graph paper are known as LINEAR scales in the same way as the scale on a foot rule is a linear scale. The scale on a foot rule may be subdivided into inches and further subdivided into tenths, eighths or sixteenths of an inch, but all the subdivisions are equal in length.

When linear scales are used on graph paper, they form a grid of squares all equal in area. This is why such graph paper is frequently referred to as "squared" paper. This type of graph paper is known as LINEAR graph paper or, in order to indicate that linear scales are being used along both x- and y-axis, the term LINEAR-LINEAR graph paper may be used.

Linear scales and graph paper have many uses and can be useful tools for the solution of mathematical, scientific, or engineering problems. There are some instances, however, where the use of linear scales is limited and where LOGARITHMIC scales have a distinct advantage. This lesson will describe logarithmic scales and the circumstances under which they can be usefully employed.

Logarithmic Scales

On a logarithmic scale the divisions, instead of being equally spaced, are made proportional to logarithms of numbers rather than to the numbers themselves. An excellent example of a logarithmic scale is that to be found on the scales C and D on a slide rule which are used for multiplication and division.

Figure 1 shows a 5-inch length of line divided linearly into 10 equal parts. The equal parts are numbered from 1 to 10, but could equally well have been 0.1 to 1.

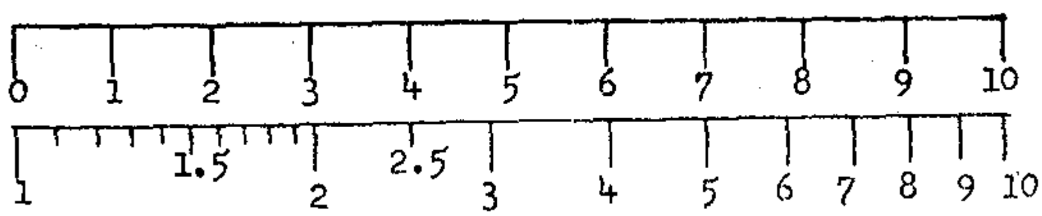


Figure 1

The scale goes from 0 to 10. Below the linear scale is shown the same length of line divided logarithmically. This logarithmic scale goes from 1 to 10 or 0.1 to 1 or 10 to 100.

Note that the logarithm of 1 (on logarithmic scale) is zero (on the linear scale). Also, log 2 (on log scale) is 0.3010 (on linear scale), log 4 (on log scale) is 0.6021 (on linear scale) and log 10 (on log scale) is 1.0 (on linear scale).

A logarithmic scale going from 0.01 to 0.1, or 0.1 to 1.0, or 1.0 to 10, etc, is said to cover or span one DECADE. A logarithmic scale can span several such decades, eg, it could go from 0.01 to 100. Such a scale would be made of 4 decades, each like the one in Figure 1, and this scale is shown in Figure 2.

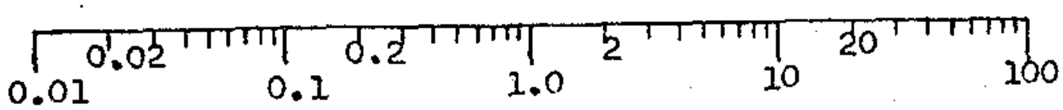


Figure 2

It can be seen from Figure 2 that each decade of the scale is subdivided in exactly the same manner. The scale in Figure 2 spans 4 decades or 4 CYCLES.

Uses of Logarithmic Scales

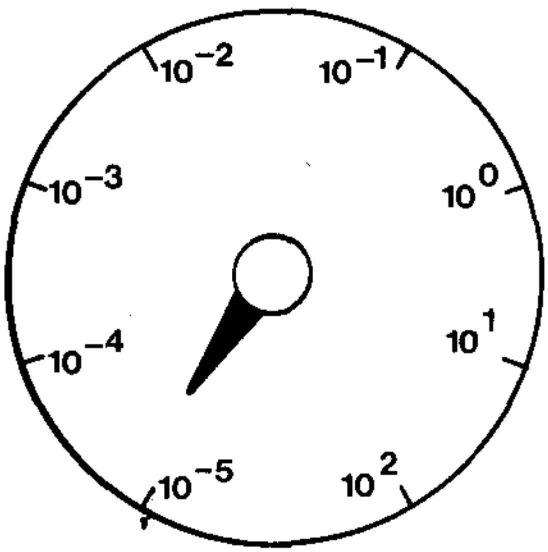
If the linear scale in Figure 1 is examined, it is clear that the distance between 0 and 1 is only 1/10 of the total length of the scale. If this distance is further subdivided into 10 equal parts, each part would be 1/1000 of the full scale value of 10, ie, each part is 0.1. Such a scale then could be used to measure to 0.1, since these subdivisions could be read with fair accuracy. However, it would not be possible to subdivide each 0.1 any further, because the subdivisions would be too small. Therefore, with a linear scale, fractional value of a measured quantity cannot be measured with any accuracy.

The same length of scale can, however, be spanned with as many decades of a logarithmic scale as is desirable. For example, the same length of scale as in Figure 1 is spanned by 4 decades in Figure 2. If the scale in Figure 2 went from 0.001 to 10, it would be easy to measure a 0.001 or 0.002 on this scale, ie, 0.01% of the full scale reading. If more decades were used, the measurement could be even smaller than this. It must be remembered, however, that the distance between 1 and 10 now only occupies the top decade and that there is, therefore, a loss of accuracy with the larger values. We can say that:

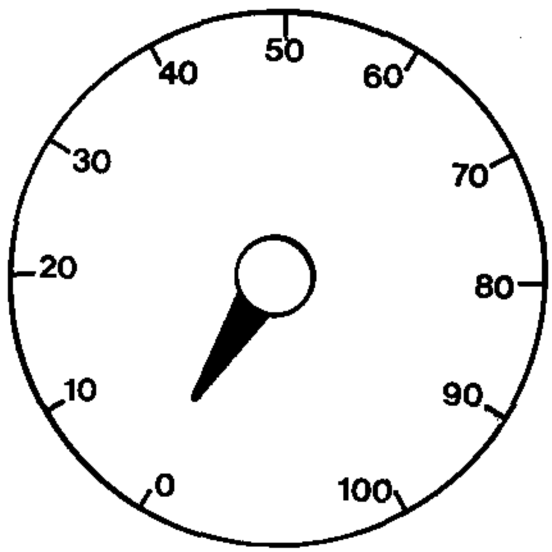
> The advantage with a logarithmic scale is that it expands the low end of the scale.

The disadvantage with a logarithmic scale is that it contracts the high end of the scale with consequent loss of accuracy.

A logarithmic scale would, therefore, be used where a large range of values are to be measured. For example, reactor neutron power may vary from full power (100%) down to zero. During normal operation of a reactor, a linear scale from 0 to 100% neutron power would be adequate. However, when the reactor is started up, reactor power may only be 0.001% or less of full power, but it is important that these low power values be measured. A guage with a scale as shown in Figure 3, is in fact used on start up from 0.001% to approximately 10% full power. Above 10%, the linear scale becomes more accurate.



% Full Power
(Log Scale)
Figure 3



% Full Power
(Linear Scale)
Figure 4

Note that low values of power, such as 0.001% and 0.01% are easily read and can be determined much more accurately than on a linear scale of the same size. However, values of power from 10% and up could not be measured as accurately as on the linear scale, ie, 92% full power could be much more accurately determined on the linear scale.

The only method of obtaining the same accuracy over the whole range of values is to use a linear scale, the range of which can be varied with some suitable range switch. In effect, this replaces one scale with a number of scales, each covering, say, 1 decade of the logarithmic scale.

Figure 5 shows another example of the use of a logarithmic scale. The radiation field in a room may normally vary from 0.1 mr/hr to 10 mr/hr, but it may well increase up to 100 mr/hr,

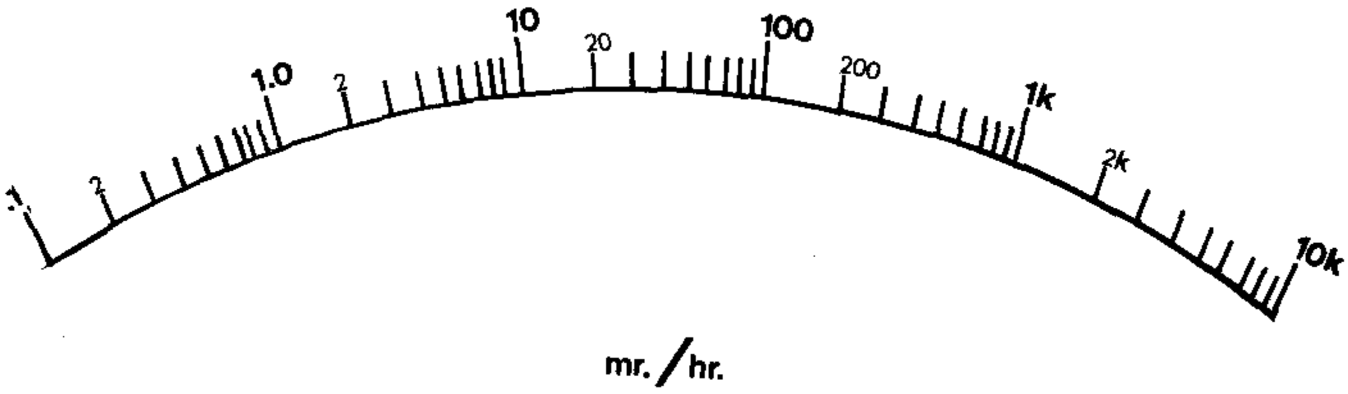


Figure 5

1000 mr/hr, or even higher. The only method of covering such a range on one scale is to use a 5-decade logarithmic scale as shown. Normal fields are clearly read and high fields can also be measured to the accuracy required. It would not be possible to say, with any certainty, whether the field was 8400 or 8500 mr/hr but such accuracy would not be required.

Logarithmic Graph Paper

Logarithmic graph paper is graph paper which is ruled with logarithmic divisions or scales instead of linear scale with the divisions all equal. There are as many different types of logarithmic graph paper as there are uses for such graph paper but they all fall into one of two main groups:

1. SEMILOGARITHMIC or LOG-LINEAR graph paper, in which the paper is ruled with a logarithmic scale in one direction (say, along the y-axis) and with equal divisions, or a linear scale in the perpendicular direction. Examples of such graph paper are shown in Figures 6 and 7.

2. LOG-LOG graph paper, in which logarithmic spacing is used in both directions. Log-log graph paper has been used in Figure 8.

Logarithmic graph paper is further classified by the number of decades covered by the logarithmic scale. The number of decades covered is known as the number of CYCLES. Thus, 6-cycle semilog graph paper will have a 6-decade logarithmic scale in one direction and a linear scale in the other direction. A 4 x 6 cycle log-log graph paper spans 4 decades one way and 6 decades in a perpendicular direction.

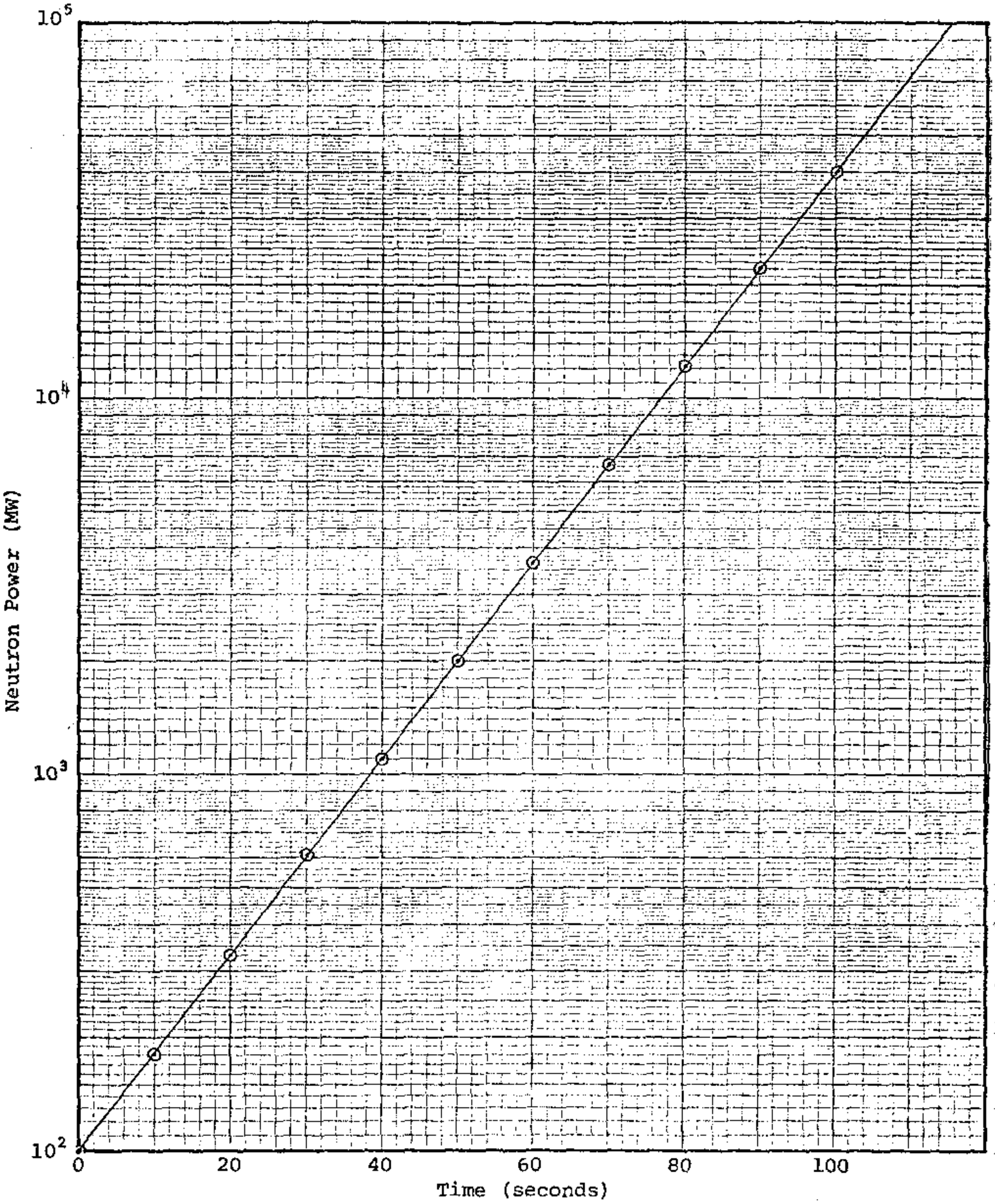


Figure 6

Reactor Neutron Power Versus Time

Gamma Dose Rate vs Penetration Depth in NPD Concrete Shield

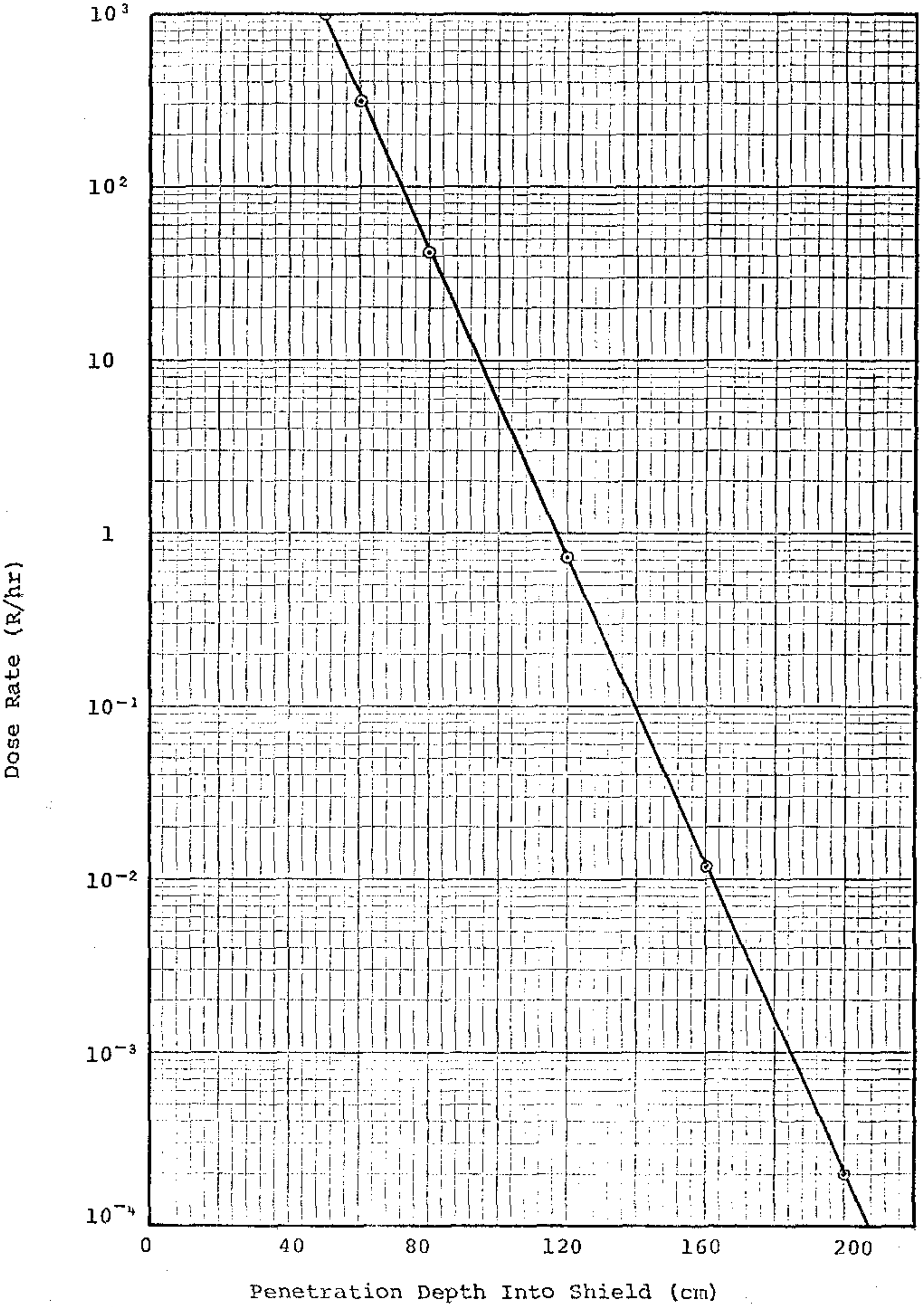


Figure 7

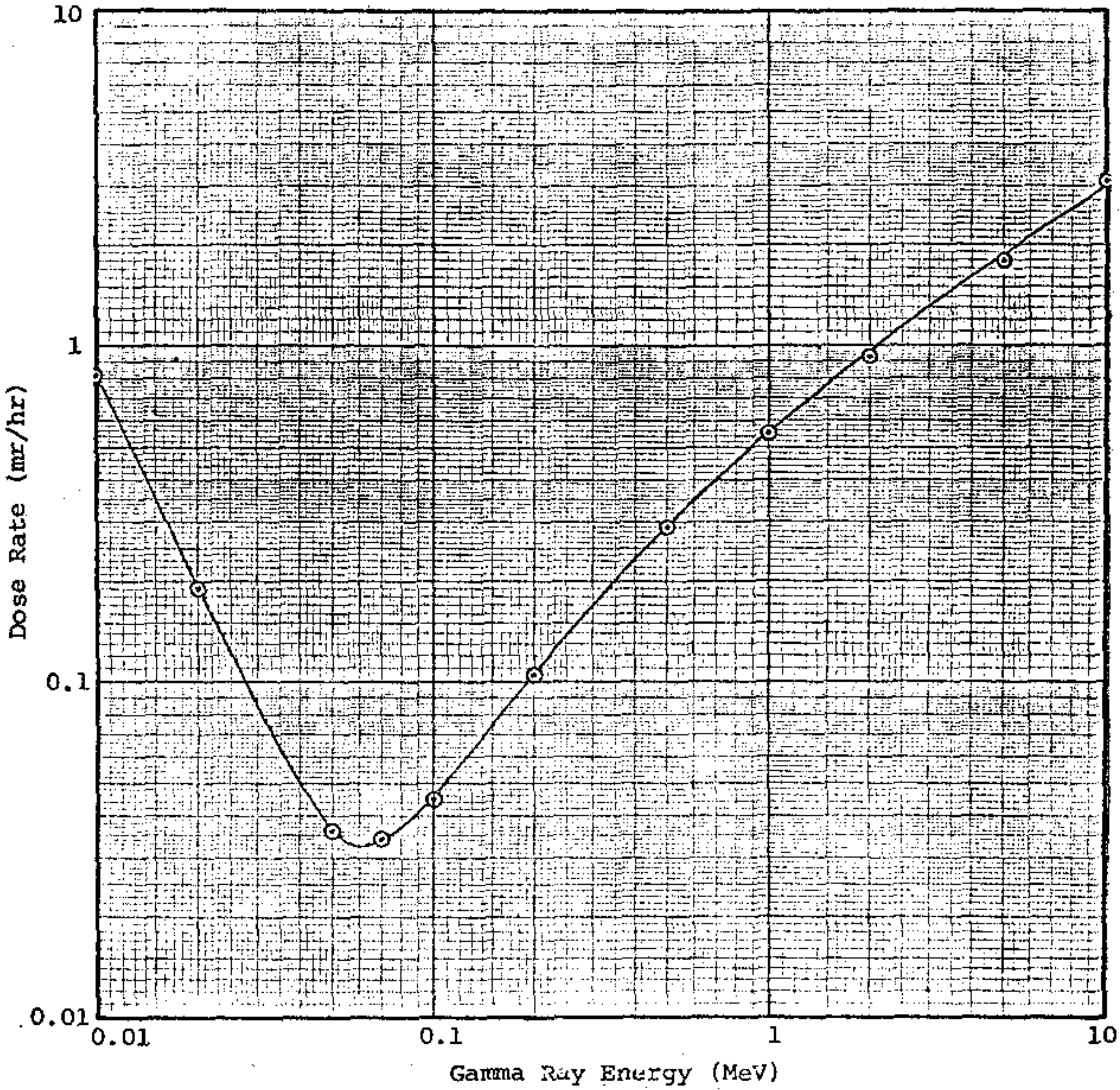


Figure 8

Dose Rate at One Meter from Gamma Source Versus Gamma Ray Energy

Uses of Logarithmic Graph Paper

The selection of graph paper for a particular purpose will be illustrated by the following examples:

Example 1:

The neutron power of a reactor, after a sudden reactivity increase, changes with time according to the equation:

$$P = 100\ e^{0.06t} \text{ Megawatts}$$

Plot the graph of the power against the time for 100 sec and determine from the graph the reactor power after 60 sec.

The calculated values of neutron power are as follows:

Time t(sec)	0	10	20	30	40	50	60	70	80	90	100
Power P(Mw)	100	182	332	605	1100	2010	3670	6650	12200	22100	40000

From the table it may be seen that a linear scale is required for the time and a 3-cycle log scale for the power. The graph is shown in Figure 6, page 5.

Power after 100 sec = 40,000 Megawatts.

Note that on semilog graph paper an exponential graph is a straight line.

Example 2:

The following gamma radiation dose rate measurements were taken at various distances through the NPD concrete shield:

Distance into shield from inner face (cm)	50	60	80	120	160	200
Dose Rate (R/hr)	1×10^3	3.1×10^2	42	0.72	1.2×10^{-2}	2×10^{-4}

The distance scale must again be a linear one but the dose rate has to be a logarithmic scale covering 7 decades. The graph is shown in Figure 7, page 6.

Since the graph is again a straight line, it can be concluded that the gamma dose rate decreases exponentially through the shield.

If the acceptable radiation dose rate outside the shield is 1 mr/hr or 1×10^{-3} R/hr, a shield thickness of 184 cms, or just over 6 ft, would have been sufficient.

Example 3:

The dose rate, at a distance of 1 meter from a source of 1 millicurie, varies with the energy of the gamma rays emitted by the source. The following table shows the dose rate for various energy gamma rays. Plot the curve of dose rate against gamma energy and estimate the energy when the dose rate is a minimum.

Gamma Energy (MeV)	0.01	0.02	0.05	0.07	0.1	0.2	0.5	1.0	2.0	5.0	10.0
Dose Rate (mr/hr) at 1 meter from source	0.82	0.19	0.036	0.034	0.045	0.105	0.29	0.55	0.93	1.8	3.1

Both quantities span 3 decades and so we require 3 x 3 cycle log-log graph paper. The graph is shown in Figure 8, page 7.

From the graph, the dose rate is a minimum when energy = 0.062 MeV.

Example 4:

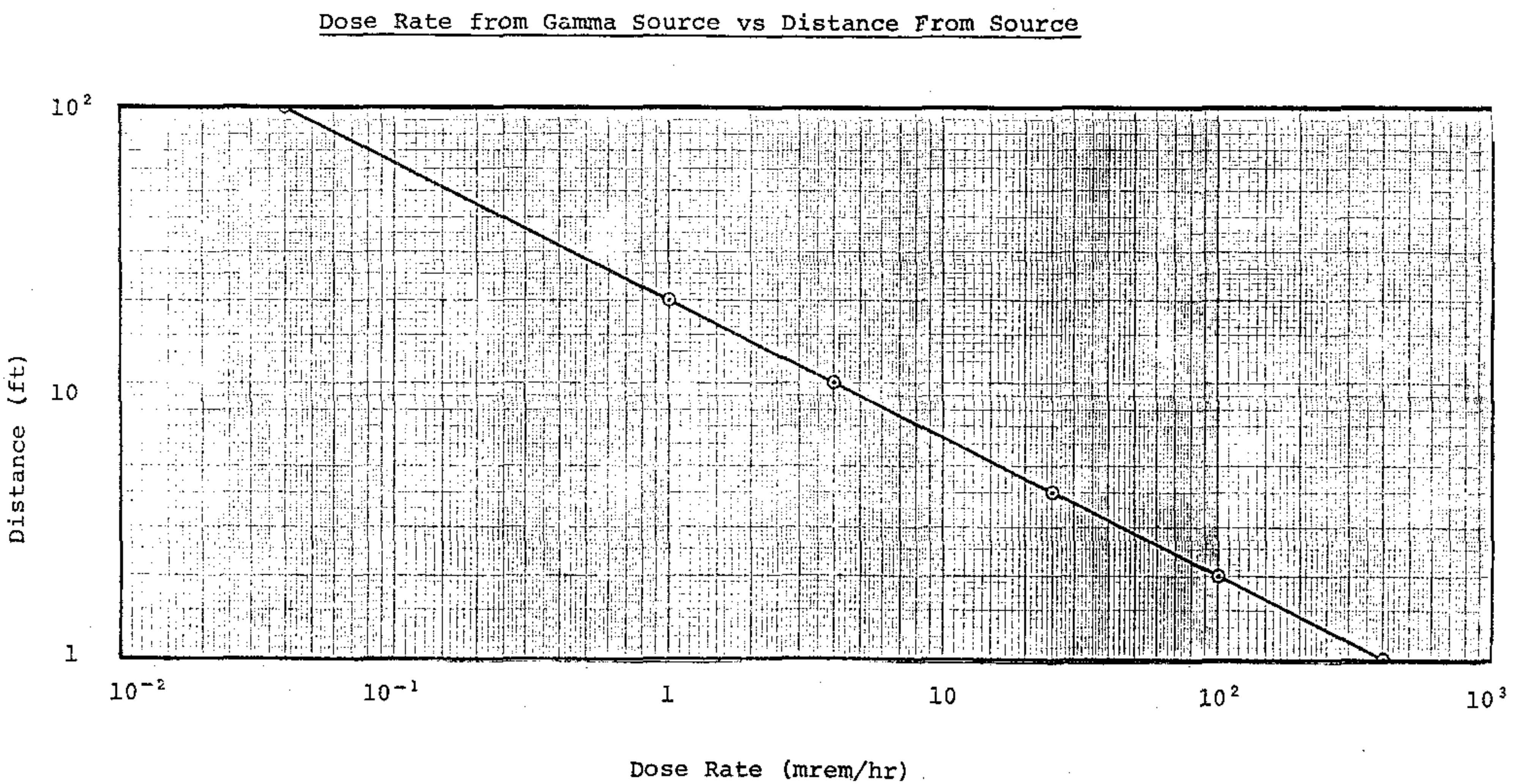
The radiation dose received in one hour from a small gamma source varies inversely with the square of the distance from the source. Consider a gamma source which causes an exposure of 400 millirems per hour at a distance of one foot. At other distances, the dose rates can be found by using the inverse square law. A few calculated values follow:

Distance (ft)	1	2	4	10	20	100
Dose Rate Millirems/hr	400	100	25	4	1	0.04

Plotting this graph on log-log paper has two advantages:

1. A wide range of values can be covered.
2. The curve becomes a straight line. (See Figure 9, page 10)

If the student will try to plot a graph of the above information on a linear-linear graph sheet, he will immediately see the difficulties involved.



Dose Rate from Gamma Source vs Distance From Source
Distance (ft)
10²
10
1
10⁻²
10⁻¹
1
10
10²
10³
Dose Rate (mrem/hr)

Figure 9

ASSIGNMENT

1. (a) What is the basic difference between the divisions on a logarithmic scale and a linear scale?

   (b) What is a decade on a logarithmic scale?

2. State the advantage and disadvantage of a logarithmic scale over a linear scale.

3. Under what circumstances would a logarithmic scale be used?

4. The following table shows the decrease in neutron power in a reactor following a trip.

Neutron Power (% Full Power)	100	2.2	1.0	0.3	0.058	0.013	0.0028	0.0013	0.001
Time (Minutes)	0	0.5	1.0	2	4	6	8	10	12

Plot the graph of neutron power against time and determine from the graph the time required for the neutron power to decrease to 0.1% of full power.

5. The total weight of heavy water in the air in the boiler room of a nuclear electric station required to produce a certain tritium concentration is given in the following table.

Tritium Concentration (M.P.C.)	100	500	1000	5000	10000	50000
Weight D_2O (lb)	1.62	8	16.2	80	162	800

Show graphically how the tritium concentration varies with the weight of heavy water in the room. From the graph determine the tritium concentration when there are 25 pounds of D_2O in the air in the room

6. The thermal power in a reactor following a reactor trip varies with time as shown in the following table.

Time (seconds)	0	0.5	1	5	10	100	1000	10,000
Thermal Power (% full power)	100	92	67	12.2	7.5	3.9	2.2	1.25

Plot the graph of thermal power against time and, from the graph, determine how long it takes for the power to drop to 6% of full power.

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