Instrumentation & Control - Course 136

INTRODUCTION TO FREQUENCY RESPONSE

The problem of assessing the performance response of a control system to changes in input and load is difficult. A closed loop system may be unstable and subject to oscillation, not only does this make useful measurement impossible but it may well be damaging. The answer to this is to investigate the system with the loop opened, in which case it will be stable, and from these results make a decision on the closed loop behaviour.

The question is therefore, can we predict the transient response of a system by looking at the open loop characteristic? Frequency Response testing is such a technique. The system response to a range of sinusoidal inputs is measured and the transient response is estimated from these results. This estimation is often quite crude but it is the practicality of the method that makes it a powerful tool.

It must be stated that real signals applied to the system will not be sinusoidal but sinusoids are used as a practical means of evaluation.

First Order Response

A first order device can be considered as a single capacity system. If the input to a first order system is changed the output will immediately begin to indicate that change. The rate of change will depend upon the capacitance of the system.

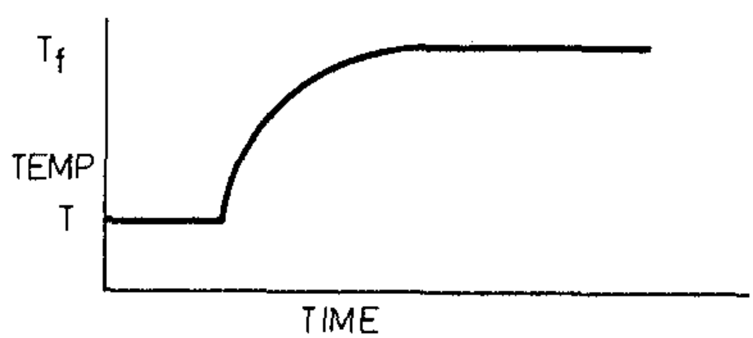


Figure 1: First Order Response to a Step Change.

Consider a device or system, which can be described as first order, being subject to a variable frequency, constant amplitude input. The response of the device can be seen from observation of the phase and amplitude of the output.

Typical first order devices include, thermocouples, transmitters and I/P transducers. Another example would be to cycle the inflow to a tank to cause a variation of the level about the set point (inflow = input, level response = output). The level should be able to follow the input for low frequency variation of inflow. There should be no phase lag and no loss of amplitude at low frequencies. As the frequency of the cycling input is increased the tank level is less able to respond to the inflow changes so that the level fluctuations will decrease and the phase lag will increase.

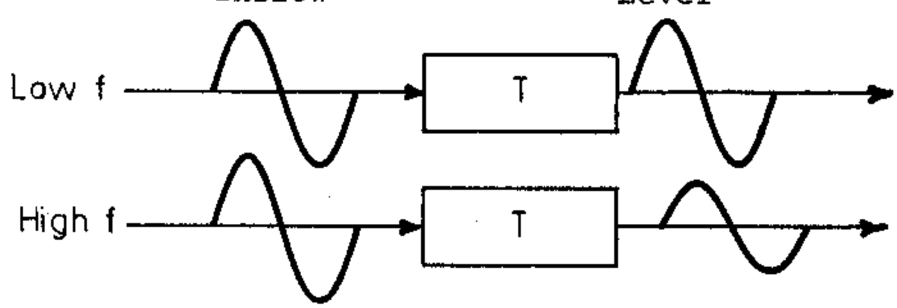


Figure 2: Response of System to Sinusoidal Inputs.

It would be advantageous to be able to describe both magnitude and phase lag mathematically. Assuming that the outflow (q_o) is laminar flow:

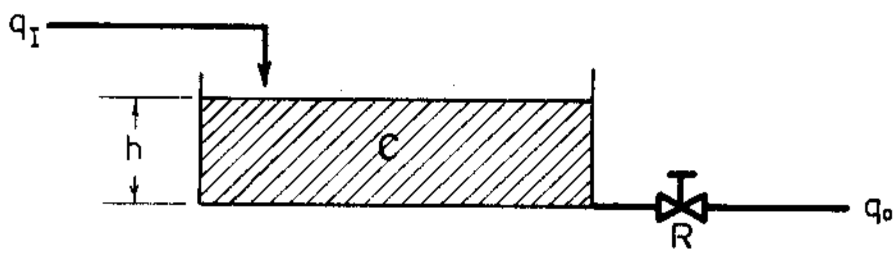


Figure 3: Open Tank System.

q_i = inflow

q_o = outflow

C = capacitance (volume/unit level)

R = hydraulic resistance

h = level

inflow - outflow = rate of accumulation

$$q_i - q_o = C\frac{dh}{dt} \qquad (1)$$

We must assume that laminar outflow results in the outflow being a function of the level ($q_o = h/R$). Substituting this relationship in place of q_o in equation (1).

$$q_i - \frac{h}{R} = C\frac{dh}{dt}$$

$$q_i = C\frac{dh}{dt} + \frac{h}{R}$$

$$Rq_i = RC\frac{dh}{dt} + h \qquad (2)$$

The quantity RC is the time constant for the system and is defined as the time required for a responsed 63.2% to an applied step change to occur. For example consider a temperature transmitter subjected to a 100°C change in temperature. The time required to indicate 63.2°C change will be the time constant 63.2°C change will be the time constant for the transmitter.

The first order equation with the time constant will be

$$Rq_i - T\frac{dh}{dt} + h \qquad (3)$$

This equation expresses the system in terms of time. We wish to consider the response of the system to a variable frequency input. To achieve this the usual method is to take the Laplace transform of this equation; this can be considered as changing an expression from a function of time to a function of frequency. The following simplified table lists the only Laplace operations that will be used in this course:

Function	Transform
$f(t)$	$L[f(t)] = \int_0^\infty e^{-st}f(t)dt = F(S)$
$f^I(t) = f(\frac{d}{dt})$	$L[f^I(t)] = sf(S)$

Taking the Laplace transform of equation (3) results in:

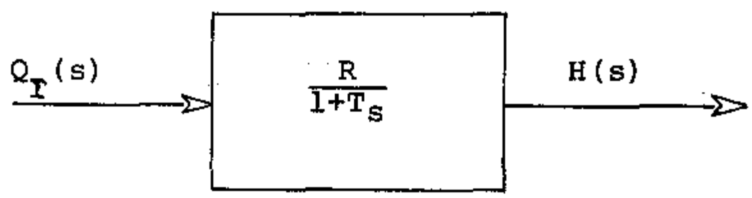
$$RL[q_i] = TsL[h] + L[h]$$
$$= L[h](Ts + 1)$$

Our prime interest is in the ratio between output and input. This is more usually, in control theory, referred to as the transfer function.

For this example the output will be the level (h) and the input the inflow (q_i).

$$\frac{L[h]}{L[q_i]} = \frac{R}{1 + T_s}$$

This can be shown in block diagram form as:



Consider the ratio of level to inflow (input to output) as a function G(S) which describes the given application when the hydraulic resistance is unity (R = 1).

$$\text{then,} \quad G_s = \frac{1}{1 + T_s}$$

This is the general format of a first order device or system transfer function with a time constant T and a low frequency gain of one. Knowing the transfer function enables the calculation of phase lag and amplitude at a particular frequency to be undertaken.

First Order Lag

The phase lag is the angular difference at some point in time, between the cycling input and the corresponding cycling output of the system.

$$\tan(\phi) = wt \qquad \text{where, } G(S) = G(jw)$$

$$\phi = \tan^{-1}(wt) \text{ (lag)}$$

G(jw) is a complex number, treated as a vector quantity, the modulus of which is the steady state sinusoidal response and the phase angle is the phase shift of the system.

First Order Amplitude Ratio

The amplitude ratio is the ratio of the output amplitude to the input amplitude, ie:

$$\frac{X}{X_o} = \frac{1}{\sqrt{1 + (WT)^2}}$$

where, X = output amplitude

X_o = input amplitude

W = frequency in radians per second

T = first order time constant

These formulae can be applied to any device which approximates a first order response. Rather than express the amplitude ratio as a decimal value it is more usually stated in decibels (dB) and designated the magnitude ratio (M).

$$M = 20 \log_{10} \left(\frac{X}{X_o}\right) \text{ dB}$$

Using decibels rather than decimals will improve the readability of the Magnitude Ratio scale.

Example: What is the magnitude ratio in decibels when the amplitude ratio in decimal form is 0.33.

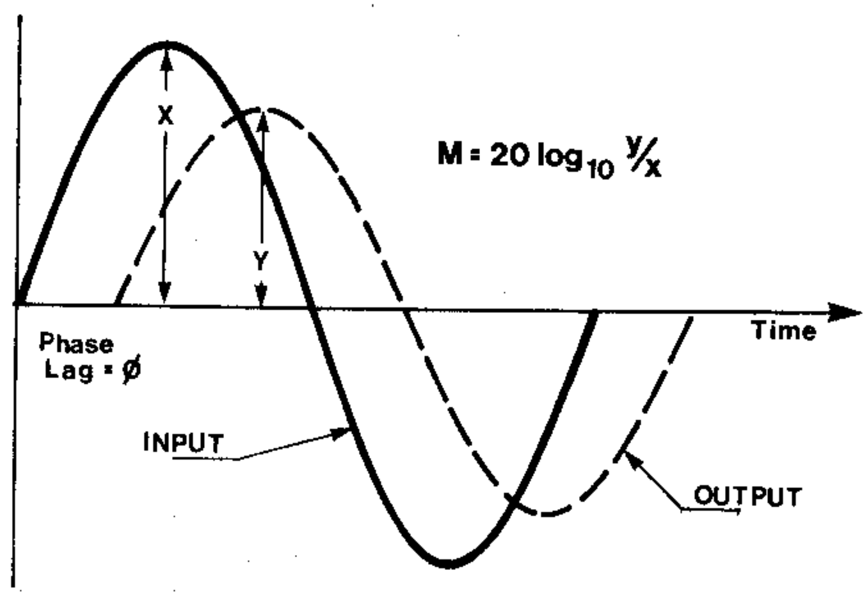
$$M = 20 \log_{10} \left(\frac{X}{X_o}\right) = 20 \log_{10} (0.33)$$

$$= 20 (\bar{1}.5185)$$

$$= 20(-.4815)$$

$$= -9.63 \text{ dB}$$

For any system it is possible therefore to calculate the magnitude ratio and phase lag at any particular frequency as shown in the diagram.



To evaluate a system fully in such a way would require a large number of individual plots and it is therefore much simpler to plot a graph of magnitude and phase shift against a base of frequency. Such a graph is called a Bode Plot.

Example - Plot the magnitude ratio and phase lag Bode curves for a temperature transmitter which approaches a first order response with a 2 second time constant.

$$G(jw) = \frac{1}{1 + 2(jw)}$$

Table M and ϕ values when w = 0.01, 0.1, 0.5, 1 and 10 rads/sec.

Solution

$$w = 0.01$$

$$\frac{X}{X_o} = \frac{1}{\sqrt{1 + (.01 \times 2)^2}} = 0.999$$

$$M = 20\log_{10} (0.999) = -.0087 \text{ dB}$$

$$= \tan^{-1} (.01 \times 2) = 1.15° \text{ lag}$$

The same method can be used for each frequency point resulting in the following table.

W	M(dB)	ϕ
.01	-.008	-1.15°
.1	-.175	-11.3°
.5	-3.0	-45°
1.0	-7.0	-63.4°
10.0	-26.0	-87.1°

Plotting these values will result in the Bode Plot shown. If lines are drawn tangent to the magnitude curve along the horizontal and sloped curves they will intersect at what is called the corner frequency. The reciprocal of the corner frequency will be the time constant of the system. For a first order response the magnitude ratio will always be -3 dB at the corner frequency. For a first order system the response is usually considered flat until the corner frequency is reached and it will then drop off at 20 dB per decade or 6 dB per octave above the corner frequency.

Straight Line Approximation (First Order System)

For a first order system it is often sufficient to approximate the Bode plot by straight line methods.

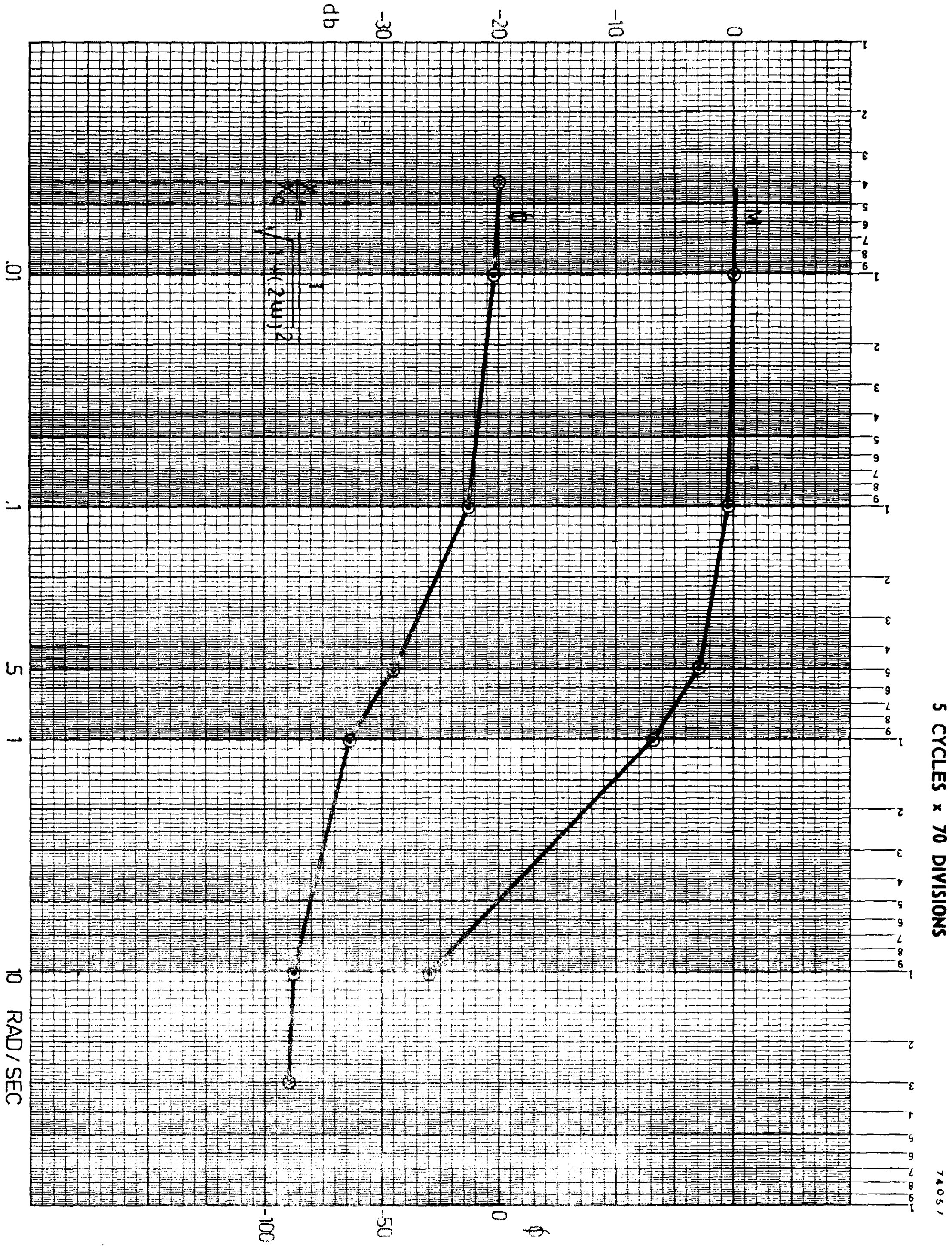
M Curve

1. Locate the corner frequency (equal to reciprocal of Time constant (I/T)).

$$\frac{X}{X_o} = \frac{1}{1 + (wt)^2} = \frac{1}{1 + (T/T)^2}$$

$$M = 20 \log_{10} \frac{1}{2} = -3 \text{ dB}$$

$$= \tan^{-1}(wt) = \tan^{-1} = 45°$$



5 CYCLES x 70 DIVISIONS
db
0
-10
-20
-30
M
φ
.01
.1
.5
1
10 RAD/SEC
φ
0
-50
-100
74057

2. Sketch the magnitude curve along the 0 dB line (or the stated low frequency magnitude) to the corner frequency and then slope down at 20 dB/decade.

3. Smooth the M curve to include the -3 dB point. Return to the straight line approximation 1 octave up and 1 octave down from the corner frequency

   1 octave up = 2 x wc

   1 octave down = wc/2

ϕ Curve

4. Use a straight line approximation with 45° lag at the corner frequency, 0° lag 1 decade down, and 90° 1 decade up from the corner.

   1 decade up = 10 x wc, 1 decade down = wc/10.

5. At 1 decade below the corner frequency, the phase lag will be approximately 6°.

$$\phi = \tan^{-1} \left(\frac{0.1T}{T}\right) = \tan^{-1} (0.1) = 5.7°$$

6. At 1 decade above the corner frequency the phase lag will be approximatly 84°.

$$\phi = \tan^{-1} \left(\frac{10T}{T}\right) = \tan^{-1} 10 = 84.3°$$

7. Smooth the phase curve about the straight line to include the 6°, 45° and 84° points.

   Example sketch the Bode M and ϕ curves for the following transfer function:

$$G(jw) = \frac{1}{1 + 0.1(jw)}$$

   Solution: The transfer function describes a first order system.

   Corner Frequency = 1/T = wc (T = 0.1)

   wc = 10 Rad/sec

   M = 0 dB until wc, and will then drop at 20 dB/decade

   M = -3 dB @ wc, ϕ = 45° @ wc

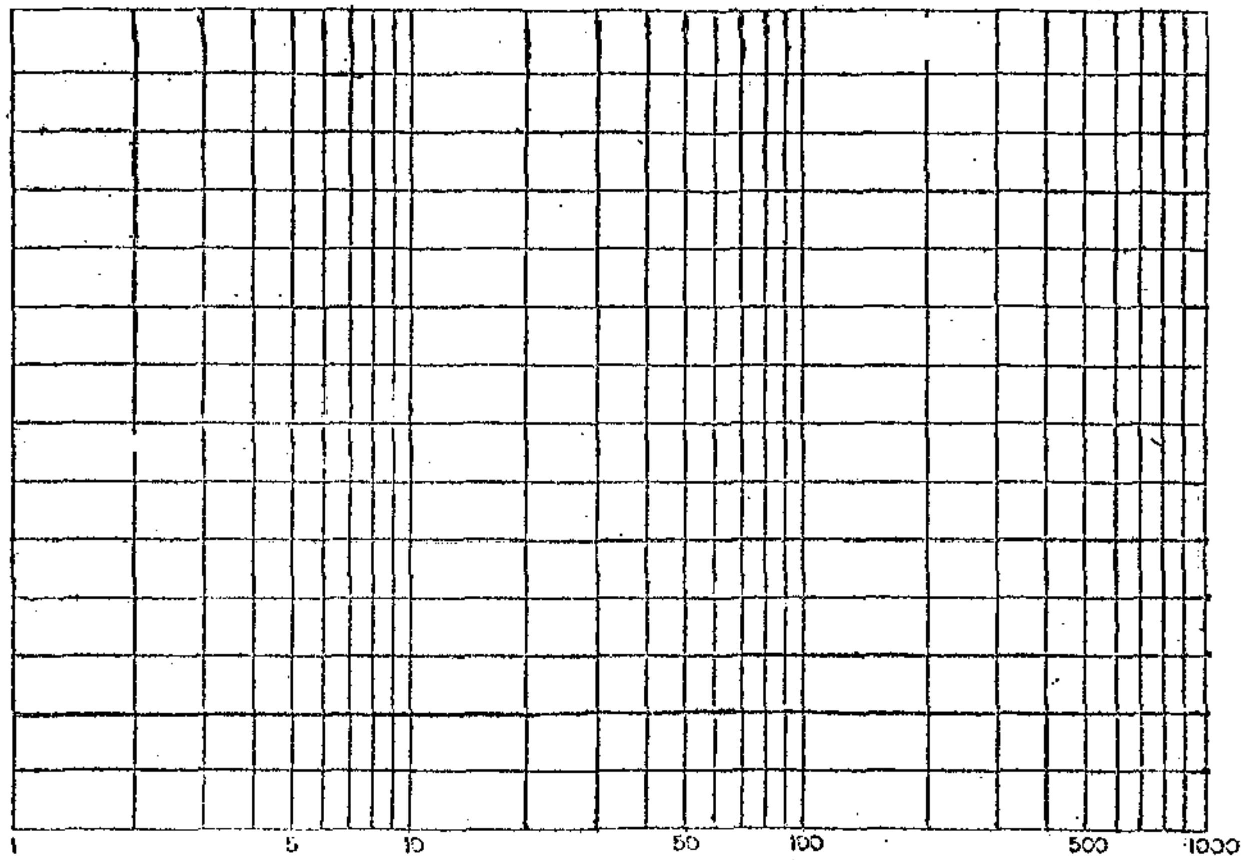
   ϕ = 6° @ 1 Rad/sec (0.1 wc)

   ϕ = 84° 100 Rad/sec (10 wc)

Plot the curves of the transfer function,

$$G\ (jw) = \frac{1}{1 + 0.1(jw)}$$

on this graph paper.



Analysis of Bode Plots

Bode plots can be constructed in a similar manner for complete control systems. Recall that a condition of marginal stability exists when the total loop gain is one and the phase lag is 180°. Controller settings must be adjusted to ensure the system does not approach the marginal stability condition.

The control objective is to correct a wide range of disturbances as quickly as possible without danger of instability. This can be best achieved by using the highest possible proportional gain (narrowest PB) whilst ensuring that the 180° lag comes at a high a frequency as possible to ensure stability. Stable control is provided if there is an adequate margin between the 180° phase lag frequency and the frequency where the overall loop gain is one.

Problem

Consider a pure single capacity system (a single capacity system requires all the components in the loop, except the tank, to be capable of instantaneous response - if not, the system is multi-capacity). What sort of control modes would be specified and what would the settings be? (Consider the reaction curve formulae in 136.00-3).

Solution

A pure single capacity system will exhibit a first order response. The maximum phase lag possible with a first order system is only 90° at high frequency. The total lag of 180° cannot be achieved so it is not possible for the system to sustain oscillations. The controller gain can be infinite (0° PB) and stable control is possible with no offset.

Clearly we are dealing with a hypothetical system. Pure first order systems are, to all intents and purposes, not possible. A real system will consist of several sub-systems exhibiting, at least, first order response characteristics. The overall system response will certainly be higher than first order and a phase shift of 180° will usually be achievable.

Under "real" conditions therefore we must define the stability criteria. The terms used are gain margin and phase margin.

Gain Margin

The gain margin is the difference in gain, expressed in dB, between a loop gain of one (0 dB) and the loop gain which occurs when the phase lag is 180°.

Example: The magnitude ratio value may be -6 dB at a phase lag of 180°. The gain margin in this instance will be:

GM = 0 - (-6 dB) = 6 dB

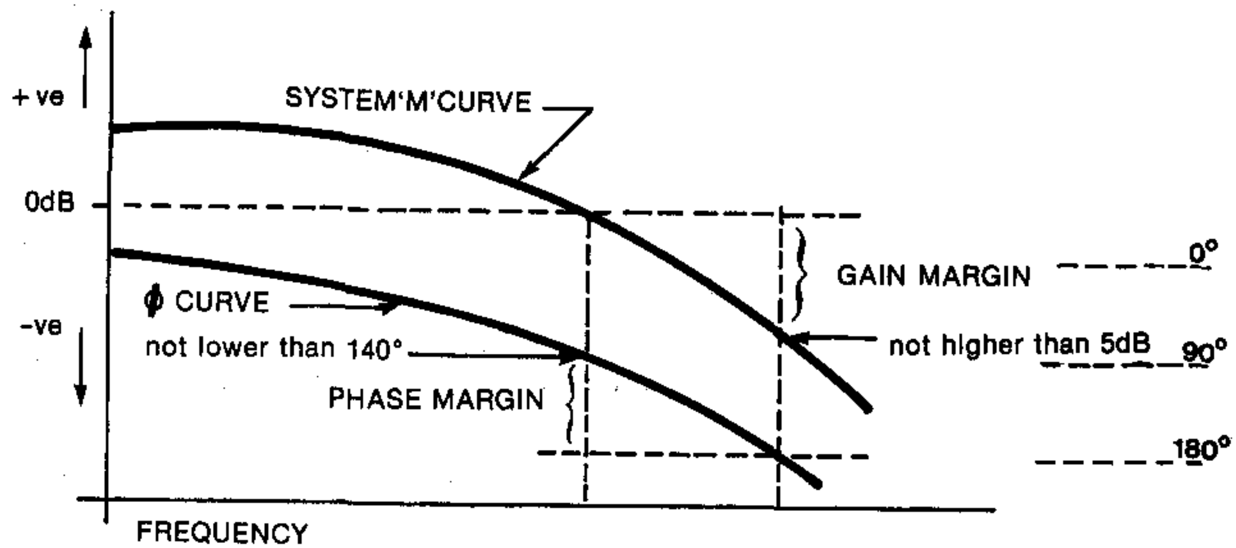


Figure 6: Bode Plot of Stable System.

Gain Margin >5 dB
Phase Margin >40°

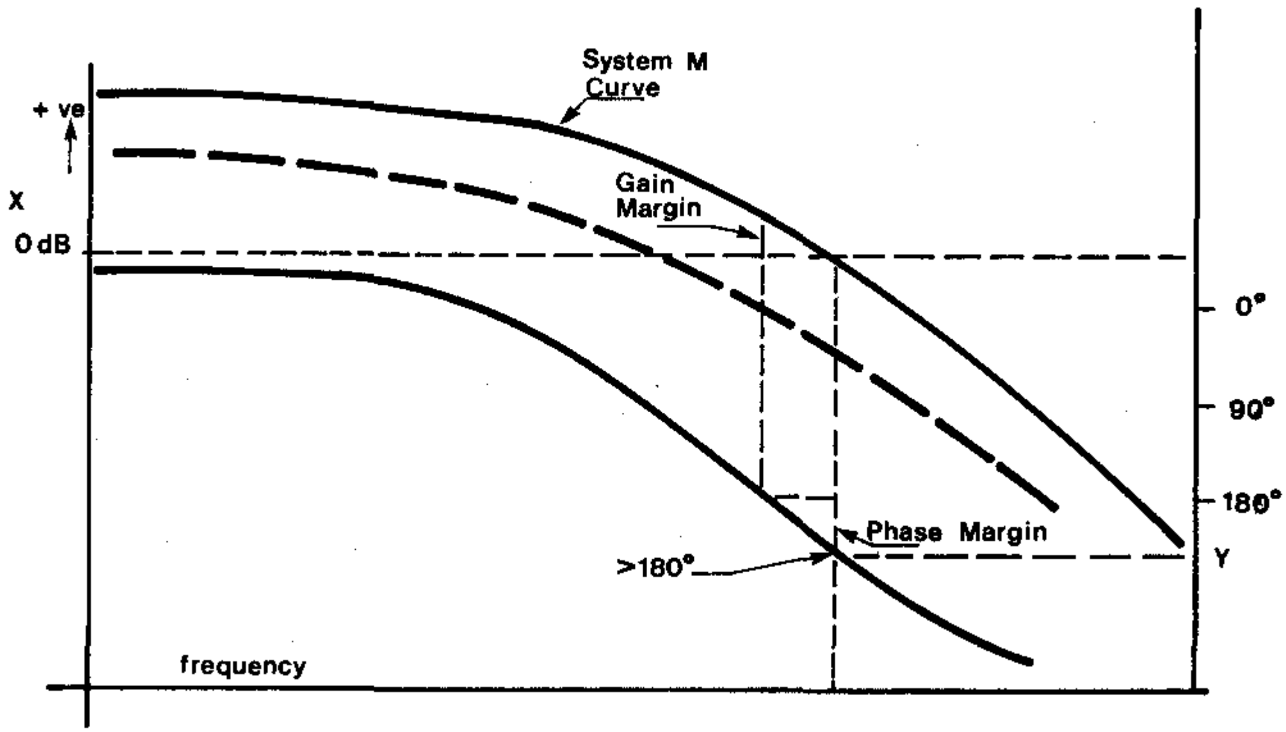


Figure 7: Bode Plot of Unstable System.

Gain Margin - 0 - xdB = -xdB (unstable)
Phase Margin - 180° - y° = - (y - 180°) unstable

Clearly, for the example quoted, at the 180° phase lag frequency the overall gain is less than 1, ie, the system is stable.

The gain margin is positive for negative decibel values. A negative gain margin denotes instability. Typical design value gain margins would be 5 - 10 dB.

Phase Margin

The phase margin is the difference between the phase lag of 180°, and the lag which occurs when the magnitude is 0 dB (Gain = 1).

Example: If the phase lag of a system is 145° when the loop gain is one, the phase margin will be:

$$PM = 180° - 145° = \underline{35°}$$

Clearly this indicates a stable condition since, as gain falls with increasing phase shift, at the 180° point the gain will be less than one.

The phase margin is positive for phase lags less than 180° (stable system) and negative for phase margins greater than 180°.

To make the system shown in Figure 7 stable, it will be necessary to lower the proportional gain (increase the PB) until the preferred value of phase margin is achieved (shown by dotted line). This adjustment of gain will not affect the phase shift.

Problem

Sketch the M and ϕ curves for the system transfer function given. This transfer function represents the proportional control of a system which is approximated by three first order time constants of 0.5, 0.05 and 0.005 seconds. Determine the value of controller gain (k) that is required to produce 45° phase margin. Initially set K = 1.0.

$$G(jw) = \frac{1}{(1 + 0.5(jw))(1 + 0.05(jw))(1 + 0.005(jw))}$$

Solution

Corner frequencies will occur at the reciprocal of the three time constants, ie, 2, 20 and 200 rads/sec.

A phase margin of 45° requires a total log of 135°.

Sketch the curves using the straight line approximation method.

At the frequency when the phase margin is 45° the M curve must be shifted such that M = 0 dB when ϕ = 135°.

From the curves sketched, the 135° lag occurs at H a frequency of 20 rad/sec and the corresponding m curve value is -23 dB.

Check: Calculate the phase lag at 20 rad/sec.

$$\phi = \tan^{-1} (0.5 \times 20) + \tan^{-1} (0.05 \times 20) + \tan^{-1} (0.005 \times 20)$$

$$\phi = 84.3° + 45° + 5.7° = 135°$$

Calculate the magnitude ratio at 20 rad/sec.

$$\frac{X}{X_o} = \frac{1}{\sqrt{(1 + (0.5 \times 20)^2)}} \cdot \frac{1}{\sqrt{(1 + (0.05 \times 20)^2)}} \cdot \frac{4}{\sqrt{(1 + (0.005 \times 20)^2)}}$$

$$= \frac{1}{(10.04)(1.41)(1.004)}$$

$$\frac{X}{X_o} = 0.07$$

$$\therefore M = 20 \log_{10} 0.07 = -23 \text{ dB}$$

To give phase margin of 45°, (ie, gain = 1 at phase lag of 135°) M curve must be shifted up by 23 dB.

Loop gain must therefore equal one.

Present loop gain is 0.07.

$\therefore$ for loop gain of one controller gain must be increased. Such that 0.07 kc = 1

$$\therefore \quad kc = \frac{1}{0.07}$$

kc = 14.29

Decibels and Magnitude Ratio

DB	MAG. RATIO M GAIN	MAG. RATIO M LOSS
.1	1.01	.989
.2	1.02	.977
.3	1.03	.966
.4	1.05	.955
.5	1.06	.944
.6	1.07	.933
.7	1.08	.923
.8	1.10	.912
.9	1.11	.902
1.0	1.12	.891
1.1	1.13	.881
1.2	1.15	.871
1.3	1.16	.861
1.4	1.17	.851
1.5	1.19	.841
1.6	1.20	.832
1.7	1.22	.822
1.8	1.23	.813
1.9	1.24	.803
2.0	1.26	.794
2.1	1.27	.785
2.2	1.29	.776
2.3	1.30	.769
2.4	1.32	.759
2.5	1.33	.748
2.6	1.35	.741
2.7	1.37	.734
2.8	1.38	.724
2.9	1.40	.716
3.0	1.41	.708
3.1	1.43	.698
3.2	1.44	.692
3.3	1.47	.683
3.4	1.48	.676
3.5	1.50	.668
3.6	1.51	.661
3.7	1.53	.652
3.8	1.55	.646
3.9	1.57	.639
4.0	1.58	.631
4.1	1.60	.623
4.2	1.62	.617
4.3	1.64	.609
4.4	1.66	.603
4.5	1.68	.595

DB	MAG. RATIO M GAIN	MAG. RATIO M LOSS
4.6	1.70	.589
4.7	1.72	.584
4.8	1.74	.575
4.9	1.76	.568
5.0	1.78	.562
5.1	1.80	.555
5.2	1.82	.550
5.3	1.84	.545
5.4	1.86	.536
5.5	1.88	.531
5.6	1.90	.527
5.7	1.93	.520
5.8	1.95	.512
5.9	1.97	.508
6.0	1.99	.501
6.1	2.02	.495
6.2	2.04	.490
6.3	2.07	.483
6.4	2.09	.479
6.5	2.11	.473
6.6	2.14	.468
6.7	2.17	.463
6.8	2.19	.458
6.9	2.22	.453
7.0	2.24	.447
7.1	2.27	.442
7.2	2.29	.438
7.3	2.33	.433
7.4	2.35	.427
7.5	2.37	.422
7.6	2.40	.417
7.7	2.43	.412
7.8	2.46	.407
7.9	2.48	.403
8.0	2.51	.398
8.1	2.55	.394
8.2	2.57	.388
8.3	2.61	.385
8.4	2.63	.380
8.5	2.66	.376
8.6	2.70	.371
8.7	2.73	.367
8.8	2.76	.363
8.9	2.79	.360
9.0	2.82	.355

DB	MAG. RATIO M GAIN	MAG. RATIO M LOSS
9.1	2.85	.351
9.2	2.88	.347
9.3	2.92	.343
9.4	2.95	.339
9.5	2.98	.335
9.6	3.02	.330
9.7	3.06	.326
9.8	3.10	.323
9.9	3.13	.321
10.0	3.16	.316
10.3	3.27	.305
10.5	3.35	.297
10.7	3.43	.292
11.0	3.55	.282
11.3	3.67	.273
11.5	3.76	.266
11.7	3.84	.258
12.0	3.98	.251
12.3	4.13	.243
12.5	4.23	.236
12.7	4.33	.232
13.0	4.47	.224
13.3	4.64	.216
13.5	4.75	.212
13.7	4.86	.206
14.0	5.01	.199
14.3	5.19	.193
14.5	5.31	.187
14.7	5.45	.183
15.0	5.62	.178
15.3	5.83	.172
15.5	5.99	.167
15.7	6.11	.163
16.0	6.31	.158
16.3	6.52	.155
16.5	6.69	.149
16.7	6.82	.146
17.0	7.08	.141
17.3	7.34	.137
17.5	7.50	.133
17.7	7.69	.130
18.0	7.94	.126

DB	MAG. RATIO M GAIN	MAG. RATIO M LOSS
18.3	8.21	.122
18.5	8.43	.118
18.7	8.64	.116
19.0	8.94	.112
19.3	9.25	.108
19.5	9.46	.106
19.7	9.65	.103
20.0	10.0	.100
21.0	11.2	.089
22.0	12.6	.079
23.0	14.2	.071
24.0	15.8	.063
25.0	17.8	.056
26.0	20.1	.050
27.0	22.4	.045
28.0	25.2	.040
29.0	28.2	.035
30.0	31.6	.032
31.0	35.5	.028
32.0	40.0	.025
33.0	44.9	.022
34.0	50.2	.020
35.0	56.2	.018
36.0	63.2	.016
37.0	70.9	.014
38.0	79.5	.013
39.0	89.2	.011
40.0	100	.010
41.0	112	.009
42.0	126	.008
43.0	141	.007
44.0	158	.006
45.0	178	.006

A useful table for this type of work is the decibel and magnitude conversion table. The required decibel change is tabled with the equivalent gain (positive shift) or loss (negative shift). For the previous problem an increase in gain is 23 dB was required for the M curve. Locate 23 dB on the table and read the gain factor (Gain = 14.2) necessary to achieve this shift.

Control Mode Response

If the transfer functions for various control modes are known, Bode plots can be sketched to show the controller frequency response characteristics.

Proportional

Straight proportional control will not effect the phase lag. It can only shift the M curve up or down by narrowing or widening the band. Recall that the 180° phase reversal of the proportional controller is ignored in phase considerations as it is always present.

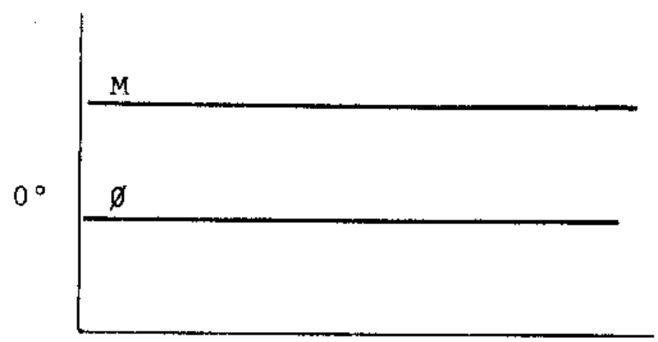


Figure 8: Proportional Control: G(jw) = k

Proportional Plus Reset

This combination can be represented by a pure integration term (1/jWT) and a first order lead term (1 + jwT). The resulting M curve will drop away from the steady state gain limit (K) at 20 dB per decade until the corner frequency is reached. The first order lead will now cancel the effect of the pure integration resulting in a flat M curve above the corner frequency. There will be a practical low frequency gain limit with a proportional plus reset controller.

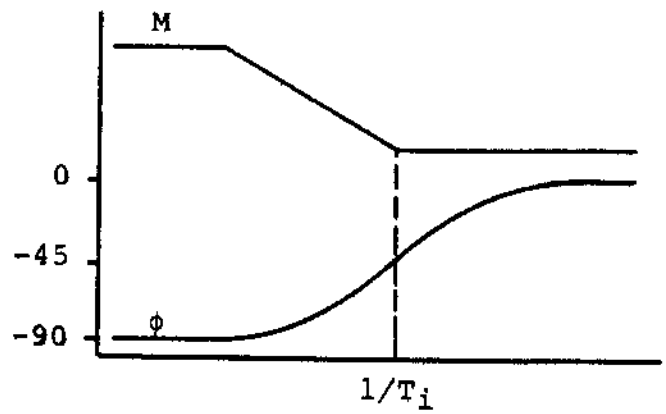


Figure 9: Proportional Plus Reset Control

$$G(jw) = K(1 + \frac{1}{jwT_i})$$

From this diagram it can be seen that reset will provide a high steady state gain at low frequency (sustained error condition) but the controller will approximate straight proportional response for high frequency disturbances.

Proportional Plus Derivative

The proportional plus derivative combination can be represented by a first order lead function (1 + jwT). The M curve will be flat until the corner frequency is reached and the first order lead will then cause a rise at +20 dB/decade until the high frequency gain limit of the controller is reached.

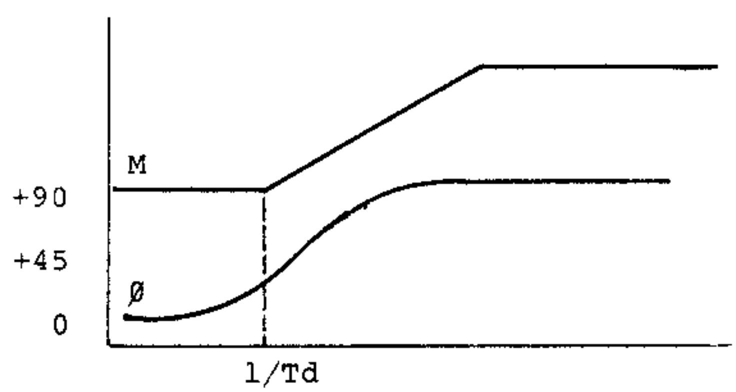


Figure 10: Proportional Plus Derivative Control k(1 + jwT).

From examining this Bode plot, it can be seen that derivative action will provide a large gain response at high frequency (sudden disturbance) but the control will approach straight proportional in the steady state.

Three Mode Controller

The three mode control response can be sketched by combining the last two Bode Plots. The M curve will drop off at 20 dB/decade until the reset corner is reached, and then the curve will be flat until the frequency reaches the rate corner. The resultant M curve will appear as a notch, the dimensions of which are dependent upon the controller settings.

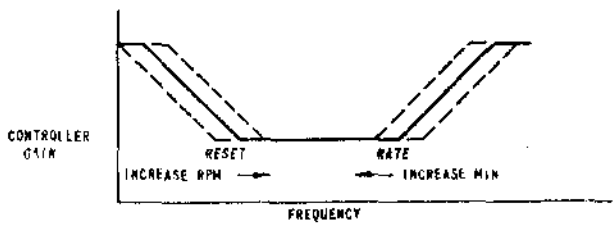


Figure 11: Three Mode 'M' Curve.

Controller Tuning

If the reset and derivative modes are minimized, ie, virtually proportional only control and therefore the notch base will be very wide. The base of the notch can be raised by narrowing the PB and until the system oscillates. This point (loop gain = 1, $\phi = 180°$) can be considered as the gain limit for the system. The band can now be widened (approximately x 2) to lower the notch base away from this frequency.

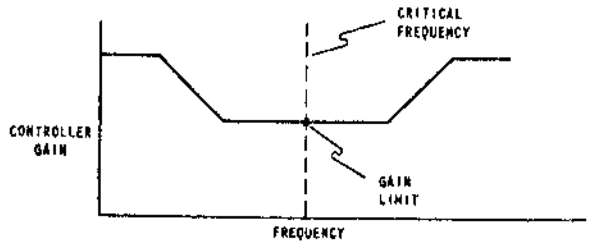


Figure 12: Adjustment of Proportional Band.

The reset rate can now be progressively increased until the system again approaches the gain limit and cycling appears. This increase in reset rate shifts the left side of the notch until it encompasses the gain limit.

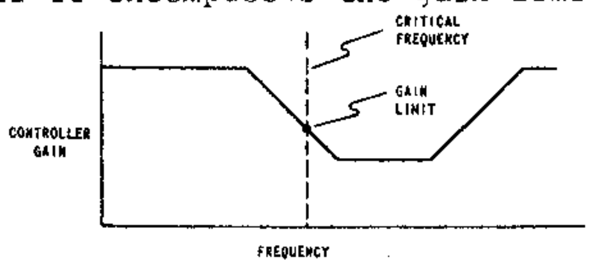


Figure 13: Adjustment of Reset Rate.

Rest rate is then decreased to ensure stability. The derivative time can be increased in small increments to improve stability and decrease the magnitude of the process deviations. This adjustment moves the right hand side of the notch towards the gain limit.

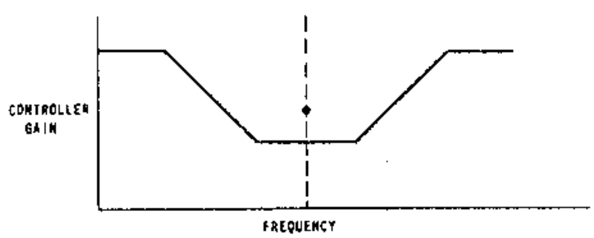


Figure 14: Tuned 3 Mode Controller 'M' Curve.

The controller is now tuned around the critical frequency of the actual system being controlled.

Derivative action (lead) will shift the 180° log frequency to a higher value. The high frequency values of the magnitude curve are also shifted upwards by the rate action.

The flatter the original system phase curve, the more benificial the addition of derivative mode will be.

Reset mode will increase the overall phase lag and the low frequency response magnitude, ie, a narrower effective frequency band.

Proportional band adjustment will only effect the M curve which can be moved up or down to provide a suitable phase margin. The gain or loss factor can be applied against an assumed gain at one to find the operating proportional band.

Control Mode Review

1. Loop components which increase lags will reduce the quality of control.
2. Too narrow a proportional band can raise the loop gain high enough to result in cycling.
3. Constant amplitude cycling is produced by a loop gain of one and a phase lag of 180°.
4. Reset action will produce high steady state gain.
5. Phase lag is increased by the application of reset but offset is eliminated. Increased reset rate can cause instability due to increased negative phase shift.
6. Derivative mode partly corrects for phase lags about the loop and will allow the use of higher gains without instability. The 180° phase lag frequency is shifted to a higher frequency.
7. Excessive derivative time will result in process cycling.
8. Derivative mode can reduce overshoot magnitude after an abrupt disturbance.
9. Open loop characteristics of a process can determine the closed loop response. (Reaction Curve Method.)

Second Order Frequency Response

To date, we have considered only the dynamics of systems that store energy in one element only - first order systems. We have seen that these systems, even with a step disturbance, respond merely with a gradual delayed output. These systems are stable, they cannot, on their own, oscillate or produce an output greater than input. Remember however that cascaded first order systems can become unstable.

If a system has more than one place to store energy, its action can not only become oscillatory but it can also produce amplification. Second order systems, those with two major storage locations are typically described by the pneumatically actuated control valve. The force operating the valve can be derived from a controller.

Consider the motion of a damped spring after being subjected to a displacing force k[f(t)].

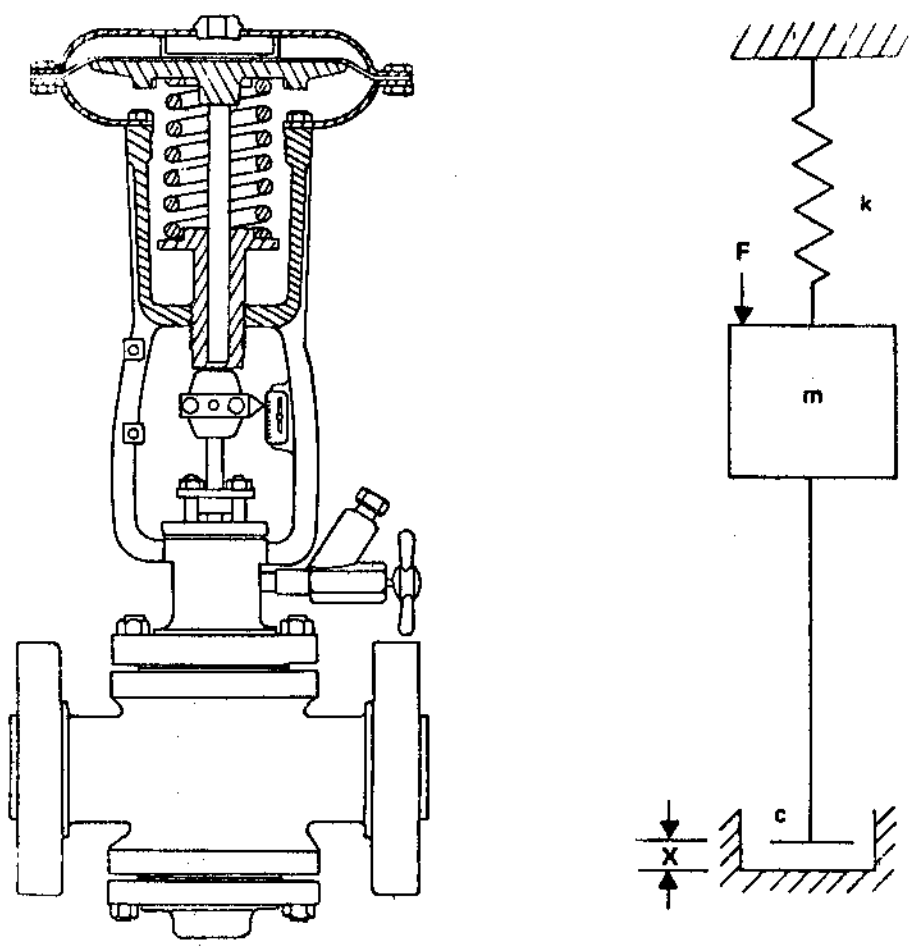


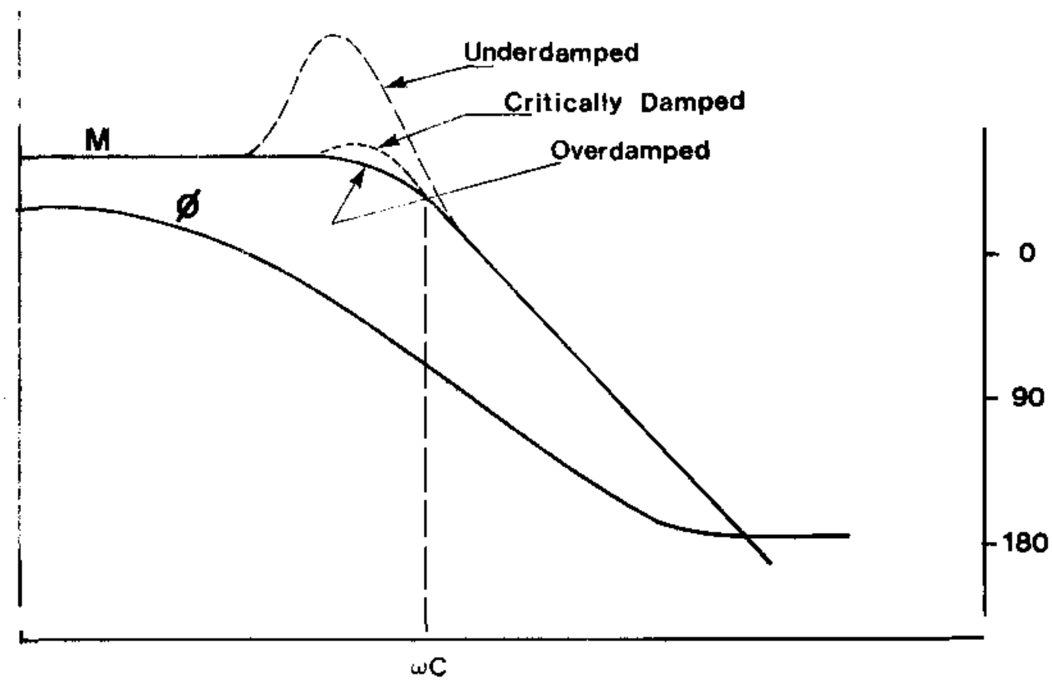
Figure 15: Spring-Mass System Example.
(a) Pneumatically-Actuated Valve.
(b) Diagrammatic Representation.

The cyclic recovery of the weight about the equilibrium point will approximate the response of a closed control loop after being subjected to a transient disturbance. The spring motion can be described by the following second order equation.

$$M \frac{d^2x}{dt^2} + R \frac{dx}{dt} + kx = k\ f(t)$$

where M = mass of the spring bob
R = Damping resistance of dash pot
k = spring constant

Depending upon the value of the damping resistance, the system can be either overdamped, critcally damped or under damped. The response curves are shown in Figure 16.



Bode Plot for 2nd Order System

It can be seen from the diagram that an under damped second order device can make effective control difficult. Second order systems, eg, control valves, should not be subjected to large step changes in signal. The phase lag for a second order system at the corner frequency is 90°, the maximum phase lag is 180° and the M curve will drop away at 40 dB per decade above the corner frequency.

ASSIGNMENT

1. The transfer function for a thermocouple can be represented as:

$$\frac{k}{1 + jwT}$$

If a particular thermaocouple has a time constant of 5 seconds, at what frequency will the signal be attenuated by 50%?

Answer: .346 Rad/sec.

2. A temperature transmitter responds as a first order device with a 10 second time constant. **Sketch** the M and ϕ Bode curves for this transmitter, labelling significant points.

3. An objective of a control system is close control without danger of instability. Explain how these conflicting objectives can be achieved.

4. Define the terms **gain margin** and **phase margin.** Use a Bode plot to aid your discussion. What is the effect on these two margins if the proportional band is widened? (M curve shifted down.)

5. A control system is represented by two first order time constants of 2 seconds and one first order time constant of 10 seconds and is under straight proportional control. What proportional band setting will provide a phase margin of 30°? (Just use straight line approximations.)

$$G(jw) = \frac{1}{(1 + 2(jw))^2(1 + 10(jw))}$$

Answer: 16%

6. Sketch a general Bode plot for a complete control system with a control gain of one. Show how the M and ϕ curves are shifted as derivative and reset are added. What is the effect of increasing or decreasing the proportional band setting?

7. Explain briefly why reset mode can decrease the stability of a given control system.

8. Consider the Bode plots of two separate systems. The first system has a relatively flat M curve and the ϕ curve drops off very sharply. The second system has a significant M curve slope but the ϕ curve slope is very gradual. Sketch these two Bode plots and determine the system for which the addition of derivative will be the most useful.

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