

Dynamics of the Subcritical Core

Neutron Sources
Subcritical Multiplication of Sources
Response Time to $+\Delta k$

What do Sources Do?

- ◆ There is measurable flux in a shut down reactor core, even if the reactor is in the Guaranteed Shutdown State (GSS), with very large amounts of neutron absorbing stuff in the core.
- ◆ This is caused by subcritical multiplication of source neutrons.

Why is there Flux in a Shut Down Reactor? How is it possible to get the first fission?

- ◆ A REACTOR, ONCE STARTED, CANNOT BE SHUT DOWN
- ◆ We refer to the reactor as:
 - in the operating state
 - in the shut down state
- ◆ But the Shut Down Reactor Core is not de-energized like other machines:
 - there is heat from fission product decay
 - there is flux from subcritical source multiplication

What is a Neutron Source?

- ◆ A neutron source injects a steady supply of neutrons, independent of power level, temperature, or any controllable parameter.
- ◆ CANDU reactors have two important natural sources:
 - spontaneous fission of U-238;
 - photo-neutrons released from deuterium nuclei in the heavy water by energetic γ -rays from fission product decay.

Spontaneous Fission

We will look at this source first in some detail.

The size of this source is about 10^{-14} of full power flux; $\approx 1 \text{ n}\overline{\text{cm}}^{-2} \text{ s}^{-1}$.

Spontaneous Fission of U-238

- ◆ Each CANDU fuel bundle has, initially, about 18.9 kg of natural uranium, 99.3% U-238.
- ◆ A CANDU-6 has 380 fuel channels & 12 bundles in each channel, over 85 Mg of U-238.
- ◆ This large quantity overwhelms any spontaneous fissions of other isotopes in the fuel.
 - Compare the U-235 content of fresh fuel (0.7%)
 - This is just over 600 kg of fissile fuel in a fresh core.

Spontaneous Fission Source Strength

- ◆ S.F. decay of U-238 is a very rare process.
- ◆ The Half-Life is $T_{1/2} = 8 \times 10^{15}$ years
- ◆ This results in over 1/2 million decays per second in the whole core, but,
- ◆ The neutrons are quickly absorbed, in about a milli-second, so
- ◆ At any instant in the core there are just over 1000 neutrons from spontaneous fission.

Neutron Density & Neutron Flux

- ◆ The Volume of the CANDU core, a cylinder about 6 m long, diameter 7 m, is over 200 m³
- ◆ 1000 neutrons is 5 neutrons per cubic meter!
 - Most neutrons slow down before they are absorbed
- ◆ Thermal neutron speed is about 2200 m/s
- ◆ Flux (ϕ) is (neutron density \times speed)

$$\phi = 5 \times 2200 \times 10^{-4} \spadesuit 1 \text{ cm}^{-2} \text{ s}^{-1}$$

This is approximately 10^{-14} of full power flux

Why is such a small flux important?

- ◆ At first start up of a CANDU this is the only flux present. It gets the thing started.
- ◆ After a very long shut down for maintenance this may be the largest source.
- ◆ A small source becomes significant because of **SUBCRITICAL MULTIPLICATION**.
- ◆ The core is configured to maximize neutron efficiency, so some of these source neutrons cause induced fissions.

Subcritical Multiplication

- ◆ Subcritical Multiplication varies, depending on how deeply “shut down” the core is
- ◆ Even in “Guaranteed Shut-down State” (GSS), with hundreds of mk of absorber added to the core, a few source neutrons cause fissions.
- ◆ As absorber is removed, bringing the reactor closer to critical, the number of fissions caused by source neutrons increases.
- ◆ The source strength doesn’t change, but fewer neutrons are absorbed uselessly.

How is this different than a Critical Core?

- ◆ A single (one time) pulse of neutrons injected into a critical core is self sustaining:
 - there are just as many neutrons in the next generation, i.e. $k = 1$
- ◆ A single (one time) pulse of neutrons injected into a sub-critical core ($k < 1$) produces fewer neutrons in each successive generation.
- ◆ the pulse will die out, but,
 - notice that $k \neq 0$: there are some fissions
 - neutrons in successive generations come from fission, not from the source.

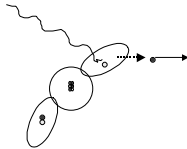
So why doesn't the flux die out with $k < 1$?

- ◆ Each "pulse" does die out, but the source continually injects new "pulses".
- ◆ There are successive generations of neutrons from fissions, induced by some of the source neutrons, so the measured flux is always larger than the source flux.
- ◆ The observed (measured) flux depends on how close the reactor is to critical:
 - deeply subcritical: each generation drops quickly
 - almost critical: many neutrons survive

A Neutron Amplifier!

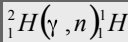
- ◆ The configuration of fuel, moderator, reflector and neutron absorber contributes to the number of induced fissions.
- ◆ The observed flux is always higher than the source flux, so we can think of the subcritical reactor as a neutron amplifier
- ◆ The source flux is the input signal
- ◆ The observed flux is the output
- ◆ The gain, as we shall show, is $\frac{1}{1-k}$

$$\frac{1}{1-k}$$



The Photo-neutron Source

The Most Important CANDU Source



The size of
this source is
about 0.03%
of full power flux

Photo-neutrons

- ◆ Some energetic γ -ray from (β^- , γ) decay of fission products (and, perhaps, activation products) interact with deuterium nuclei in the heavy water moderator and coolant, ejecting the neutrons.
- ◆ The H-2 binding energy is 2.2 MeV, so the γ -rays must have energy in excess of this.

Delayed (fission) Neutrons

- ◆ These are not really source neutrons, but they provide a useful comparison
- ◆ Some fission product (β^- , γ) decays produce excited daughter nuclei that emit an energetic neutron when they decay.
- ◆ Photo-neutrons and delayed neutrons both depend on the presence of fission products.
 - they are indirectly caused by fission
 - D.N.s are about 0.5% of the fission neutrons.
 - P.N.s strength is lower by more than an order of magnitude

How do d.n. and p.n Differ?

- ◆ The delayed neutrons show up within seconds or minutes of the fission that caused them. They affect the dynamics of power change, but not long term behaviour.
- ◆ Some photo-neutrons also show up within seconds, but many do not appear for days.
 - photoneutrons are produced in the shutdown core from fission products from previous high power operation, even months after shutdown

The Longest Lived Source

- ◆ The longest lived photo-neutrons so far positively identified come following the decay of Ba-140, with a decay half life of 12.8 days. After 3 months this source strength drops by a factor $(1/2)^{91/12.8} = 0.007$
- ◆ Data from CANDU reactors with long shut down times suggest an additional longer lived source at very low concentration.

Relative Strength

- ◆ Suppose the full power flux in the fuel is about $1 \times 10^{14} \text{ n cm}^{-2} \text{ s}^{-1}$
- ◆ This is made up, approximately, as follows
 - prompt fission neutrons 99,434,000,000,000
 - delayed fission neutrons 534,000,000,000
 - photo-neutrons 32,000,000,000
 - spontaneous fission neutrons 1
 - TOTAL 100,000,000,000,000
- last digits uncertain. Actual values depend on fuel burnup

Subcritical Multiplication

$$k < 1$$

- ◆ In a subcritical core with sources, at any instant there are present in the core:
 - the source neutrons injected in this generation
 - the fission neutrons from the source neutrons in the previous generation (source $\times k$)
 - fission neutrons from fission neutrons from the source injected two generations ago (source $\times k^2$)
 - and all fission neutrons left from previous generations

Amplification

- ◆ Adding together all the neutrons from this generation, and all those produced by previous generations:

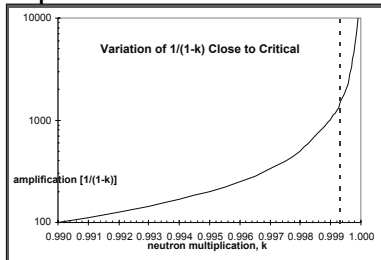
$$\phi_{\text{observed}} = \phi_{\text{source}} + k\phi_{\text{source}} + k \times (k\phi_{\text{source}}) + \text{etc.}$$

$$\phi_{\text{observed}} = \phi_{\text{source}} (1 + k + k^2 + k^3 + \dots)$$

- ◆ We saw this arithmetic in chapter 8 and we know the result is

$$\phi_{\text{observed}} = \left(\frac{1}{1-k} \right) \cdot \phi_{\text{source}}$$

Amplification Close to Critical



- ◆ This graph shows the dramatic increase in source multiplication close to critical.

Change in Power Level

- ◆ When k is increased in a subcritical reactor the source stays the same, but the number of fissions increases, so flux rises.

$$\phi_{\text{initial}} = \left(\frac{1}{1 - k_i} \right) \cdot \phi_{\text{source}}$$

$$\phi_{\text{final}} = \left(\frac{1}{1 - k_f} \right) \cdot \phi_{\text{source}}$$

$$\frac{\phi_{\text{final}}}{\phi_{\text{initial}}} = \left(\frac{1 - k_i}{1 - k_f} \right)$$

- ◆ The equation without ϕ_{source} is useful, because the source is usually not known.

Measurement of Core Reactivity

- ◆ Add a known amount of reactivity.

The difference ($k_f - k_i$) is known.

- e.g. A 15% decrease in liquid zone levels is worth 1 mk (typical: giving 100% worth 6.7 mk)

- ($k_f - k_i$) = 0.001

- Power is measured before and after

$$\frac{\phi_{\text{final}}}{\phi_{\text{initial}}} = \left(\frac{1 - k_i}{1 - k_f} \right)$$

- ◆ The flux ratio = the power ratio
- ◆ The equation can be solved for k

Measurement of Device Reactivity Worth

- ◆ Core reactivity (or k_i) can be measured, as in the preceding slide

All that is required is a method to change core reactivity by a known amount.

$$\frac{\phi_{\text{final}}}{\phi_{\text{initial}}} = \left(\frac{1 - k_i}{1 - k_f} \right)$$

- ◆ Now, insert an unmeasured absorbing rod and measure power before and after.

This is a measurement of rod worth.

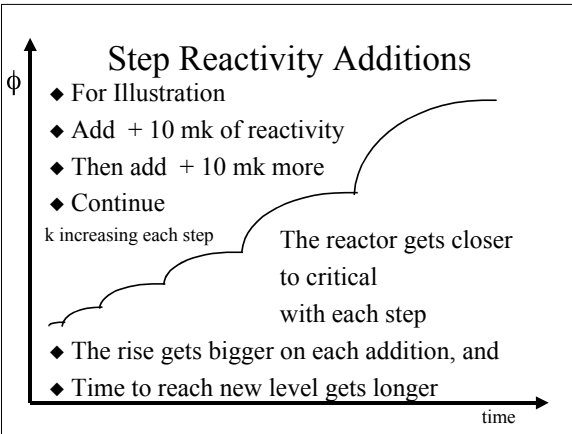
- ◆ $k_f = k_i + (\text{rod worth})$

Dynamic Response

Subcritical

Geometric Series (again)!

- ◆ $S_V = 1 + k + k^2 + k^3 + \dots = 1/(1 - k)$
- ◆ $S_n = (1 + k + k^2 + k^3 + \dots + k^n)$
- ◆ $S_n = S_V - k^{n+1} S_V$
- ◆ $\Phi_{\text{observed}} = \Phi_{\text{source}} \left(\frac{1}{1 - k} \right) - \Phi_{\text{source}} \left(k^{n+1} \left(\frac{1}{1 - k} \right) \right)$
- ◆ The last term is transient
- ◆ It “disappears” when k^n is small enough
 - for $k = 0.5$, $k^n < 0.01$ in $n \spadesuit 7$ generations
 - $1/(1-k) = 2$
 - for $k = 0.999$ $k^n < 0.01$ in $n \spadesuit 4600$ generations
 - $1/(1-k) = 1000$



Critical & Subcritical

- ◆ Add a small Δk step:
 - When subcritical, but very close to critical, there is a large power rise that flattens out after a long time
 - When slightly supercritical, the initial response is almost the same, but eventually the power begins to rise exponentially
 - and can reach any level if not stopped
- ◆ Chapter 14 will look at the transition from subcritical to critical

What is Critical?

- ◆ Theoretically, criticality is $k = 1.000\ 000 \equiv$
 - this is “unmeasureable” to sufficient accuracy to be a practical definition
- ◆ Operationally, the reactor is “critical” whenever it is close enough to critical for the regulating system to maneuver power to a requested level at the requested rate
- ◆ called “*direct regulating system control*”
- ◆ Station operating rules may allow
 - e.g. 5% - 10% zone level subcritical operation
 - this is typically about $\Delta k = - 0.3$ mk to $- 0.7$ mk

How Long does the Power Rise take?

- ◆ We can estimate n from $k^n < 0.01$ but to get n we need the time for one neutron generation.
- ◆ An average value is not good enough
- ◆ We must wait for the longer lived delayed neutrons to reach equilibrium
- ◆ A reasonable guess is to wait for the group 2 delayed neutrons to reach equilibrium
 - see next slide ($L \approx 0.995 l + 0.005 \times 30 \text{ s} = 0.15 \text{ s}$)
- ◆ Pick a neutron lifetime around 0.1 s or 0.2 s

6 Groups of Delayed Neutrons Data for an Equilibrium Fuelled CANDU

GROUP	DELAYED NEUTRON FRACTION % OF TOTAL OF $\beta = 0.54 \%$	HALF LIFE weightedav = 8.4 s mean lifetime 12.2 s
6	3.5%	0.2s
5	14.0%	0.5s
4	39.0%	2.2s
3	19.0%	5.7s
2	21.0%	22.0s
1	3.5%	54.2 s

CANDU Photo-neutron data

GROUP	PHOTONEUTRON FRACTION	HALF LIFE
	% OF TOTAL β_{n-1} (n)	
15	64.6%	2.5s
14	20.3%	41.0s
13	7.0%	2.40m
12	3.3%	7.70m
11	2.1%	27.0m
10	2.3%	1.65h
9	0.3%	4.41h
8	0.1%	53.04h
7	0.05%	12.815d

Note: The longest lived photo-neutron source is due mainly to the decay of the fission product Ba-140, for which about 4% of decays yield γ rays with $E > 2.23$ MeV.

The Two Group Equation for the Subcritical Core

- ◆ A constant source term can be added to the differential equations of Chapter 8
 - so a time dependent solution can be derived for subcritical power changes
- ◆ We will not do this.
- ◆ Instead we quote the formula, simplified to the “prompt jump” approximation
- ◆ And show some plotted graphs

Dynamics Equation (simplified) for the Subcritical Core

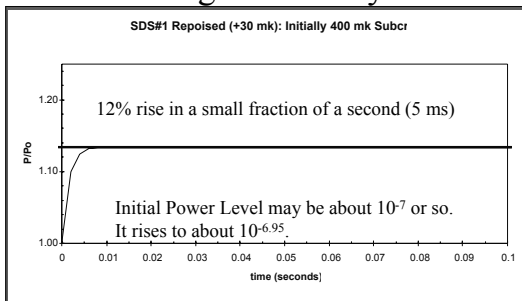
- ◆ Here, $-\rho \approx 1 - k$ in the notation of our text
 - not very accurate if $k \ll 1$: $\rho = (k - 1)/k$ is better

$$P(t) \approx \frac{P_{source}}{(-\rho_f)} \left[1 - \frac{\left(1 - \frac{\rho_f}{\rho_l}\right)}{\left(1 - \frac{\rho_f}{\beta}\right)} e^{\frac{t}{\tau}} \right]$$

Note that τ
is negative

- ◆ Use the group 2 decay constant $\lambda = 0.03 \text{ s}^{-1}$
 - formula assumes all delayed neutrons have the same λ

Response of a Deeply Subcritical Core to a Large Reactivity Addition



Response of an Almost Critical Core to a Small Reactivity Addition

