UNENE Graduate Course
Reactor Thermal-Hydraulics Design and Analysis

McMaster University
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Fuel-Coolant Heat Transfer

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Outline

- General heat conduction equation
- Heat transfer in radial direction
- Heat transfer in axial direction
- Axial and radial temperature profiles
- Convective heat transfer and boiling
- Concept of dryout
- Conduction heat transfer coefficient

General Heat Conduction Equation

- The interface between the fuel and the coolant is centrally important to reactor design because it limits the power output
- For a solid, the general energy thermal energy balance equation of an arbitrary volume

$$\iiint_{\forall} \frac{\partial(\rho e)}{\partial t}\, d\forall = \iiint_{\forall} q'''(\bar{r},t)\, d\forall - \iint_{S} \bar{q}''(\bar{r},t) \cdot \hat{n} ds$$

- Where ñ is the material density, e is the internal energy, V is the volume, S is the surface area, q''' is the volumetric heat generation, q'' is the heat flux and n is the unit vector on the surface. We replace the internal energy with temperature, T, times the heat capacity c.

General Heat Conduction Equation

- Using Gauss law, the heat balance equation is transformed into:

$$\frac{\partial(\rho c T)}{\partial t} = q'''(\bar{r},t) - \nabla \cdot q''(\bar{r},t)$$

- Using Fourier law

$$q''(\bar{r},t) = -k \, \nabla T(\bar{r},t)$$

$$\frac{\partial(\rho c T)}{\partial t} = q'''(\bar{r},t) - \nabla \cdot \, k \nabla T(\bar{r},t)$$

ρ	kg/m^3
c	J/(kg °K)
k	J/(m °K-sec)
q''	$J/(m^2\text{-sec}) = W/m^2$
q'''	$J/(m^3\text{-sec}) = W/m^3$
T	°K
α	defined as $k/\rho c = m^2/sec$.

Heat Transfer in Radial Direction (Fuel)

$$\nabla \cdot k_f \nabla T(\bar{r},t) = q'''(\bar{r},t)$$

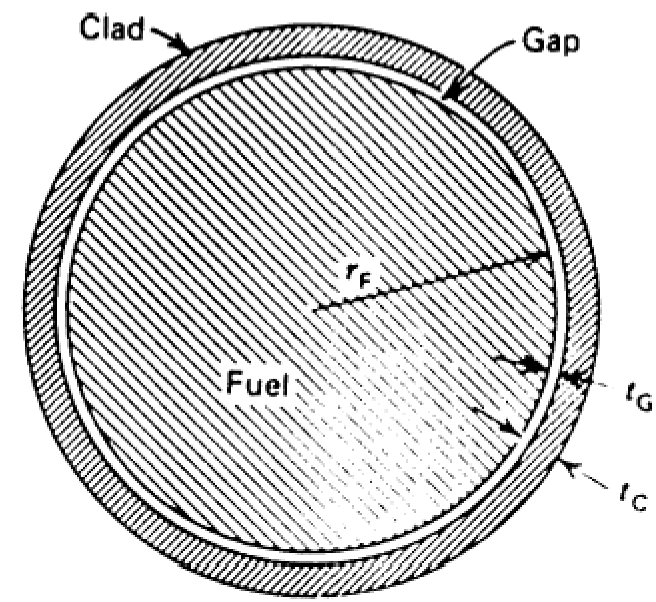
$$\frac{1}{r}\frac{d}{dr}\left(k_f\, r\, \frac{dT}{dr}\right) = q'''(r)$$

$$k_f\, r\, \frac{dT}{dr} = -\frac{r^2}{2}\, q'''(r)$$

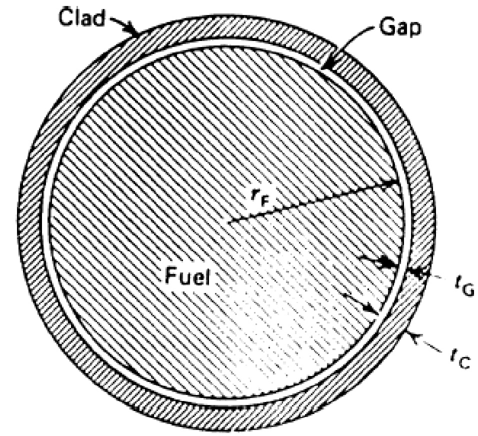
$$\int_{T_0}^{T_F} k_f(T)\, dT = -\frac{r_F^2}{4} q''' \equiv \bar{k}_f\,(T_F - T_0)$$

$$\Delta T_{fuel} \equiv T_0 - T_F = \frac{r_F^2}{4\bar{k}_f}\, q''' = \frac{q'}{4\pi \bar{k}_f}$$

where $q' \equiv \pi r_F^2\, q'''$ = linear power density



Heat Transfer in Radial Direction (Gap)



$$\frac{1}{r}\frac{d}{dr}\left(k_G\, r\, \frac{dT}{dr}\right) = 0$$

$$k_G\, r\, \frac{dT}{dr} = \text{constant}$$

$$-k_G\frac{dT}{dr}\bigg|_{r=r_F} = q'' = \frac{q'}{2\pi r_F}$$

$$k_G\, r\, \frac{dT}{dr} = -\frac{q'}{2\pi}$$

$$\therefore k_G\, \frac{dT}{dr} = -\frac{q'}{2\pi r}$$

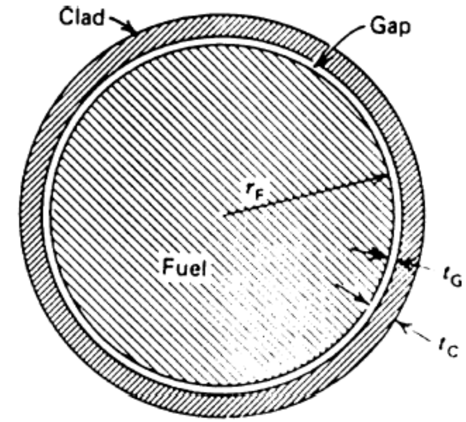
$$k_G\, \Delta T_{GAP} = k_G\,(T_F - T_C) = \frac{q'}{2\pi}\ln\left(\frac{r_F + t_G}{r_F}\right)$$

$$\therefore \Delta T_{GAP} = \frac{q'}{2\pi k_G}\ln\left(\frac{r_F + t_G}{r_F}\right)$$

$$\approx \frac{q'}{2\pi r_F}\left(\frac{t_G}{k_G}\right) \quad \text{since} \quad \ln(1+x) \approx x$$

$$h_G(\Delta T_{GAP}) = q'' \qquad \Delta T_{GAP} = \frac{q'}{2\pi r_F h_G}$$

Heat Transfer in Radial Direction (Clad)



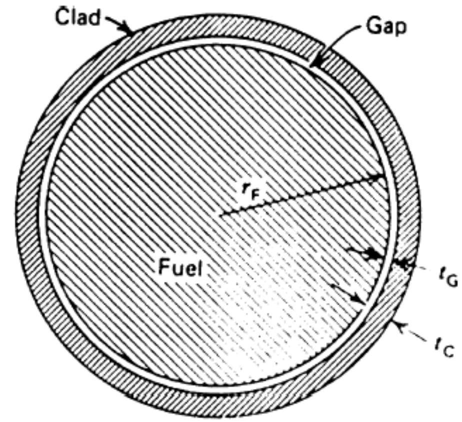
$$\frac{1}{r}\frac{d}{dr}\left(k_C\, r\, \frac{dT}{dr}\right) = 0$$

$$k_C\, \Delta T_{CLAD} = k_C\,(T_C - T_S) = \frac{q'}{2\pi}\ln\left(\frac{r_F + t_G + t_C}{r_F + t_G}\right)$$

$$\therefore \Delta T_{CLAD} = \frac{q'}{2\pi k_C}\ln\left(\frac{r_F + t_G + t_C}{r_F + t_G}\right)$$

$$\approx \frac{q'}{2\pi(r_F + t_G)}\left(\frac{t_G + t_C}{k_C}\right) \quad \text{since} \quad \ln(1+x) \approx x$$

Heat Transfer in Radial Direction (Coolant)



$$q'' = h_S\,(T_S - T_{FL})$$

$$\Delta T_{COOL} = \frac{q'}{2\pi h_S (r_F + t_C + t_G)}$$

$$T_0 - T_{FL} = \frac{q'}{2\pi}\left(\frac{1}{2\bar{k}_f} + \frac{1}{h_G r_F} + \frac{t_G + t_C}{k_C (r_F + t_G)} + \frac{1}{h_S (r_F + t_G + t_C)}\right)$$

Heat Transfer in Axial Direction

AXIAL VARIATION OF NEUTRON FLUX:

$$\Phi = \Phi_{max} \cos\left(\frac{\pi Z}{H}\right)$$

H – effective core length

AXIAL HEAT GENERATION FOLLOWS THE NEUTRON FLUX VARIATION

$$q''' = q_{max} \cos\left(\frac{\pi Z}{H}\right)$$

HEAT BALANCE ALONG A FUEL ELEMENT

$$w c_p \, dT_f = q''' A_f \, dz$$

w [kg/s] Mass flow rate

c_p [kJ/kg °C] specific heat

dT_f [°C] coolant temp rise between z and z+dz

A_c [m²] cross-section area of one fuel element

Heat Transfer in Axial Direction

$$w\, c_p \int_{T_{f0}}^{T_f} dT_f = q'''_{max}\, A_f \int_{-H/2}^{z} \cos\left(\frac{\pi z}{H}\right) dz$$

$$T_f = T_{f0} + \frac{q'''_{max}\, A_f\, H}{\pi\, w\, c_p}\left[1 + \sin\left(\frac{\pi z}{H}\right)\right]$$

$$= T_{f0} + \frac{q'''_{max}\, V_f}{\pi\, w\, c_p}\left[1 + \sin\left(\frac{\pi z}{H}\right)\right]$$

V_f [m^3] Volume of the fueled portion of the fuel element.

$$T_{f\,max} = T_{f0} + \frac{2\, q'''_{max}\, V_f}{\pi\, w\, c_p} \qquad \text{at } z = H/2$$

Heat Transfer in Axial Direction

THE HEAT BALANCE BETWEEN SHEATH AND COOLANT LEADS TO THE FOLLOWING EQUATIONS FOR SHEATH TEMP:

$$h C_c (T_c - T_f)\, dz = q'''_{max} A_f \cos\left(\frac{\pi z}{H}\right) dz$$

C_c [m] Circumference of the fuel sheath

T_c [°C] Sheath temp.

h [kJ/m² °C] Heat transfer coefficient

$$T_c = T_{fo} + \frac{q'''_{max} V_f}{\pi W C_p}\left[1 + \sin\left(\frac{\pi z}{H}\right)\right] + \frac{q'''_{max} A_f}{h C_c} \cos\left(\frac{\pi z}{H}\right)$$

Heat Transfer in Axial Direction

Maximum sheath temperature:

$$\frac{dT_c}{dz} = 0 \qquad z_{c\,max} = \frac{H}{\pi} \tan^{-1}\left(\frac{h C_c H}{\pi w c_p}\right)$$

$$= \frac{H}{\pi} \cot^{-1}\left(\pi w c_p R_h\right)$$

$$R_h = \frac{1}{hA} = \frac{1}{h C_c H}$$

Maximum fuel temperature:

$$\frac{T_m - T_f}{R\,H} = q'''_{max} A_f \cos\left(\frac{\pi z}{H}\right)$$

$$T_m = T_{fo} + \frac{q'''_{max} V_f}{\pi w c_p}\left[1 + \sin\left(\frac{\pi z}{H}\right)\right] + q'''_{max} V_f R \cos\left(\frac{\pi z}{H}\right)$$

R = Total thermal resistance across the fuel element

Heat Transfer in Axial Direction

Maximum fuel temperature:

$$\frac{dT_m}{dz} = 0 \qquad z_{m\,max} = \frac{H}{\pi} \cot^{-1}(\pi w c_p R)$$

Maximum fuel temperature:

$$T_{m\,max} = T_{f0} + q'''_{max} V_f R \left[\frac{1 + \sqrt{1+\beta^2}}{\beta} \right]$$

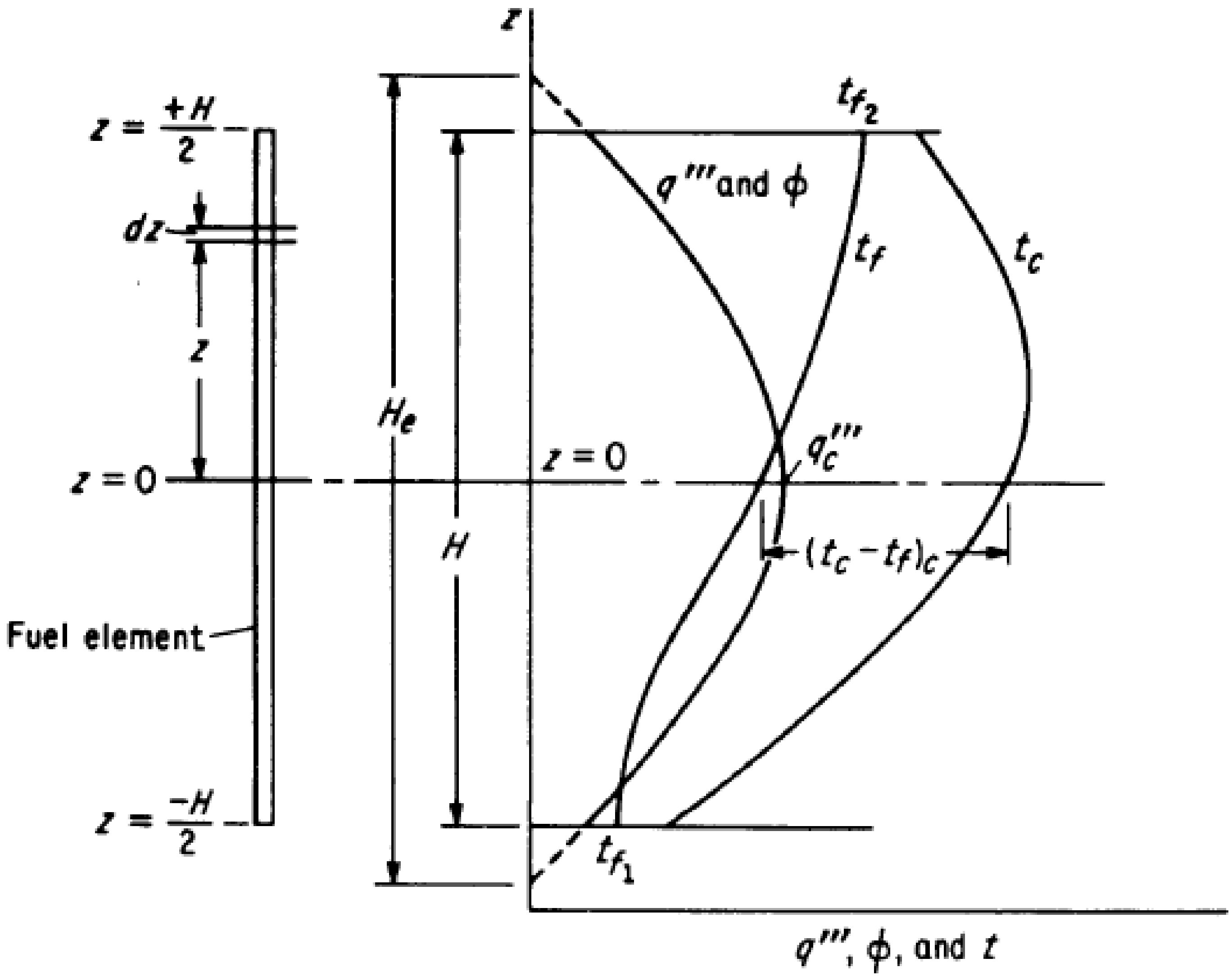
$$\beta = \pi w c_p R$$

Maximum sheath temperature:

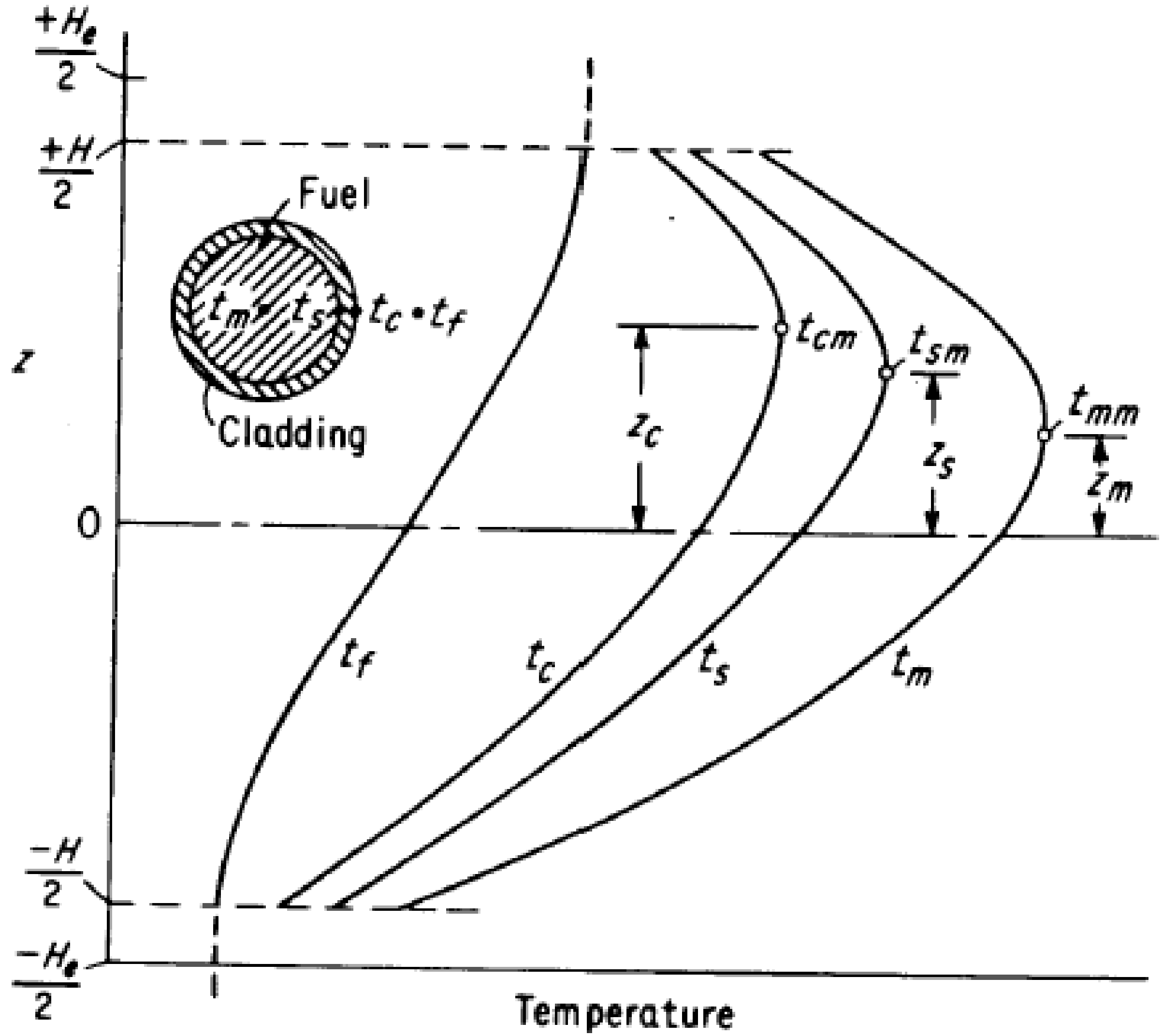
$$T_{c\,max} = T_{f0} + q'''_{max} V_f R_h \left[\frac{1 + \sqrt{1+\alpha^2}}{\alpha} \right]$$

$$\alpha = \pi w c_p R_h$$

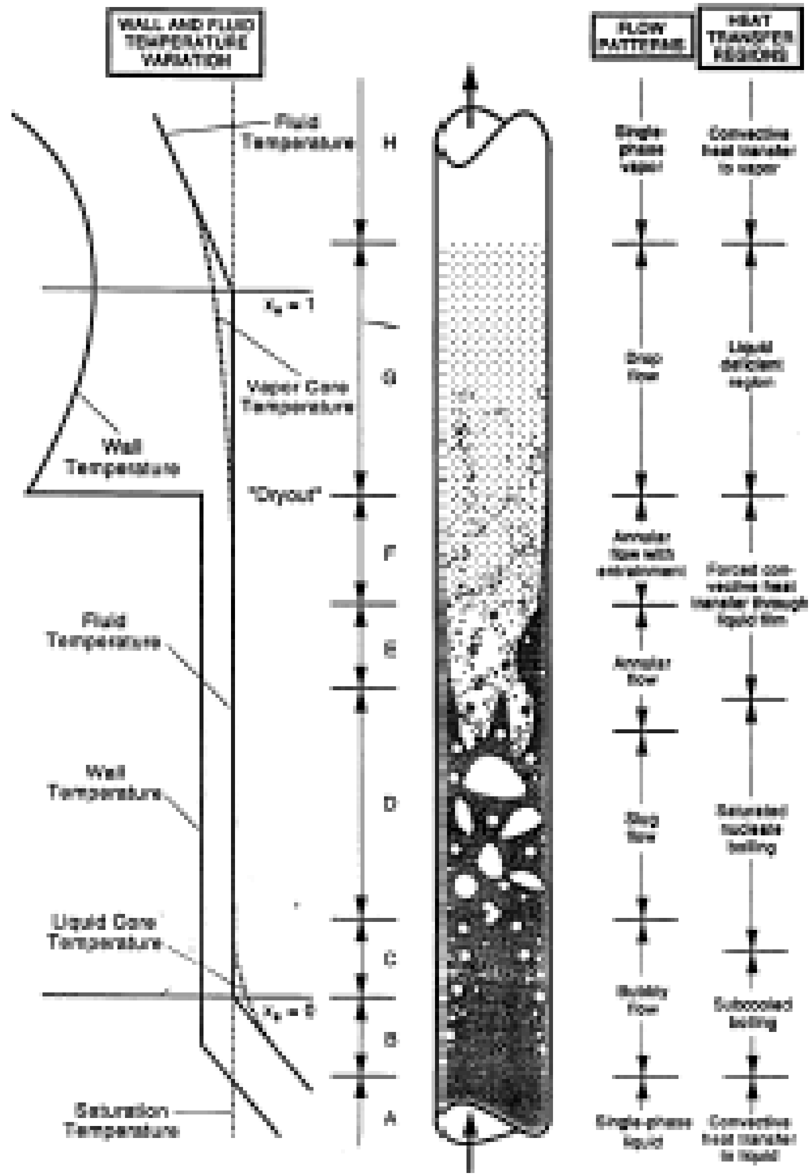
Axial and Radial Temperature Profiles



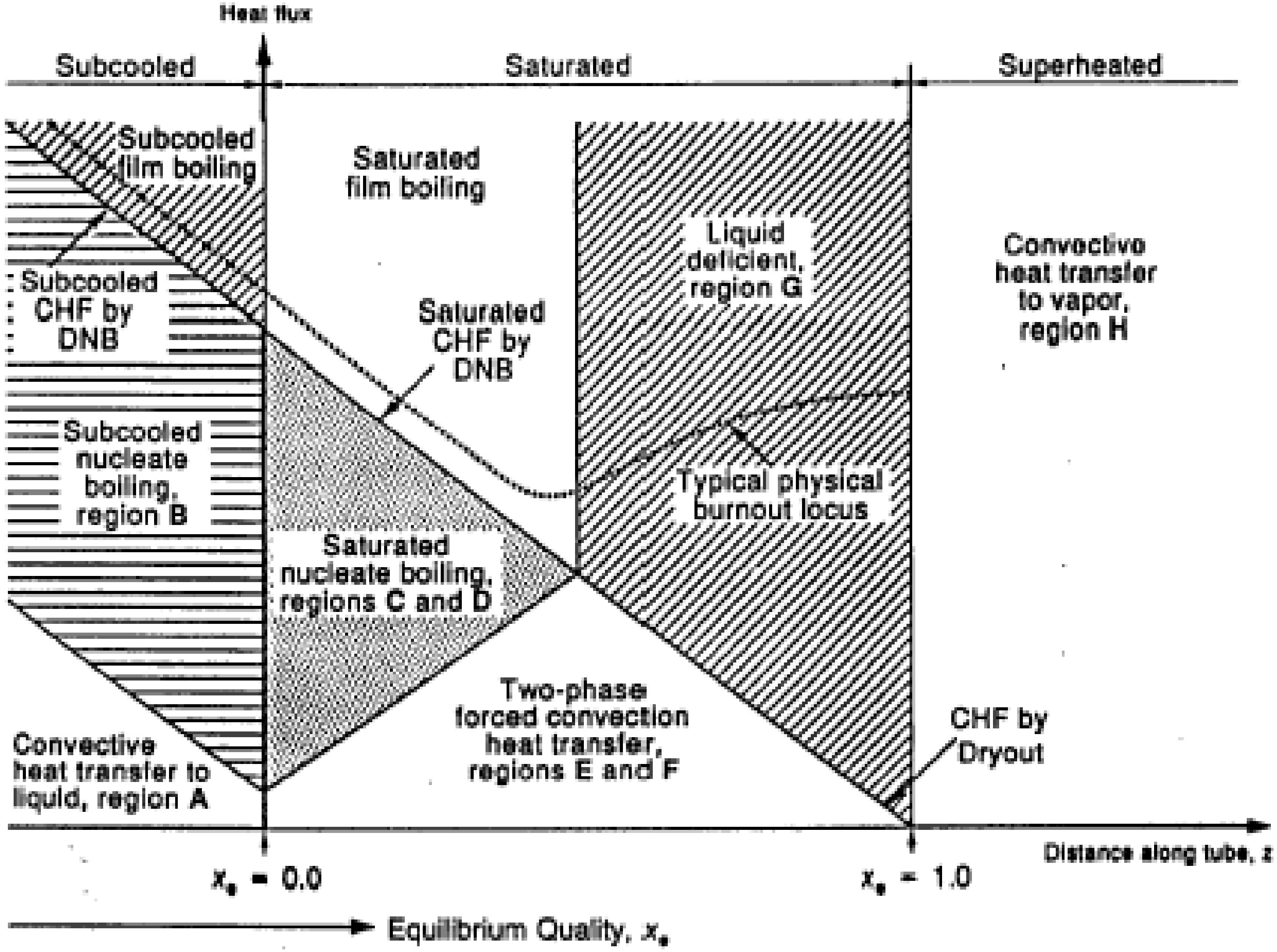
Axial and Radial Temperature Profiles



Convective Heat Transfer and Boiling



Concept of Dryout



Conduction Heat Transfer Coefficient

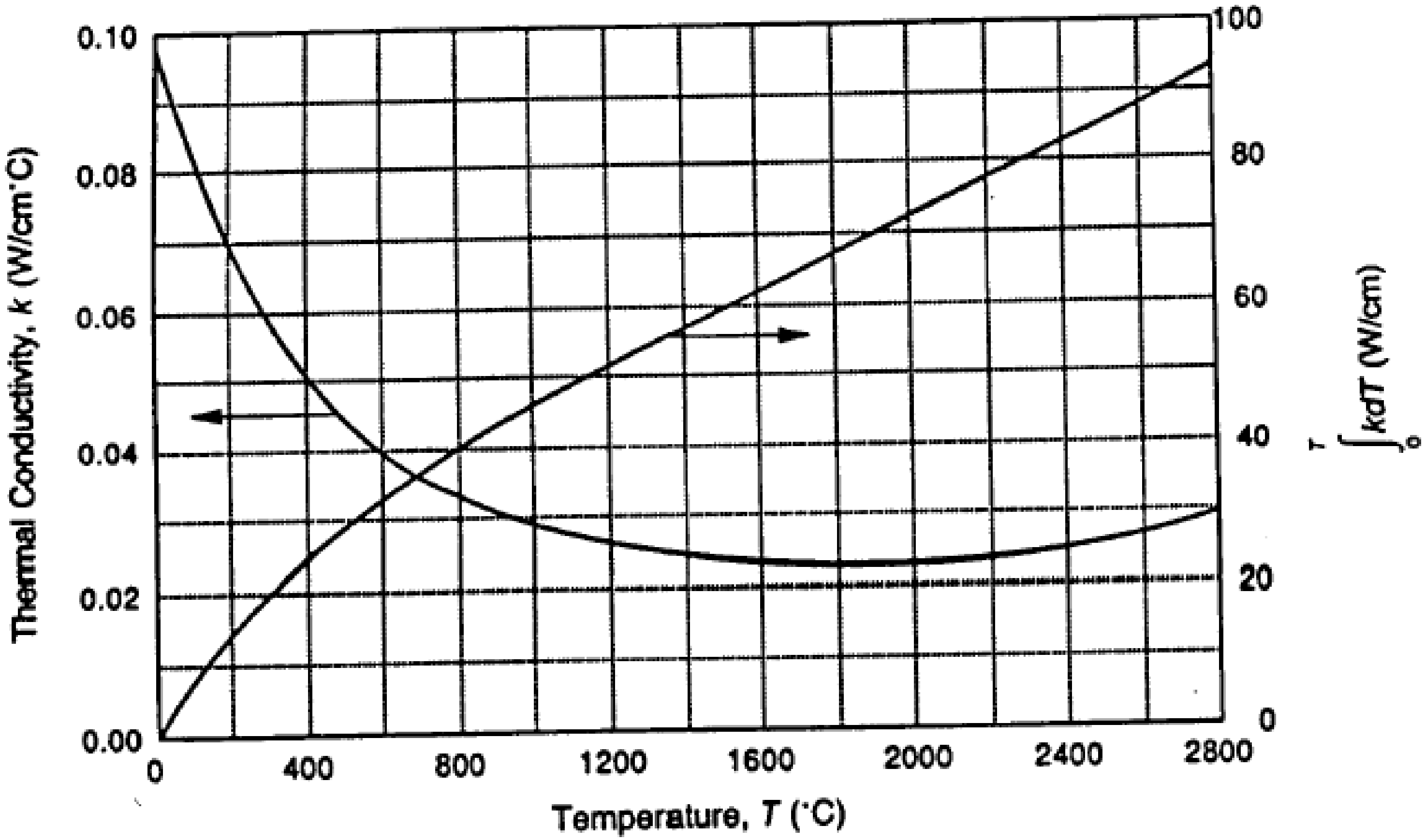


Figure 8-1 Thermal conductivity of UO_2 at 95% density from Lyon. *(From Hann et al.* [9].*)*

Conduction Heat Transfer Coefficient

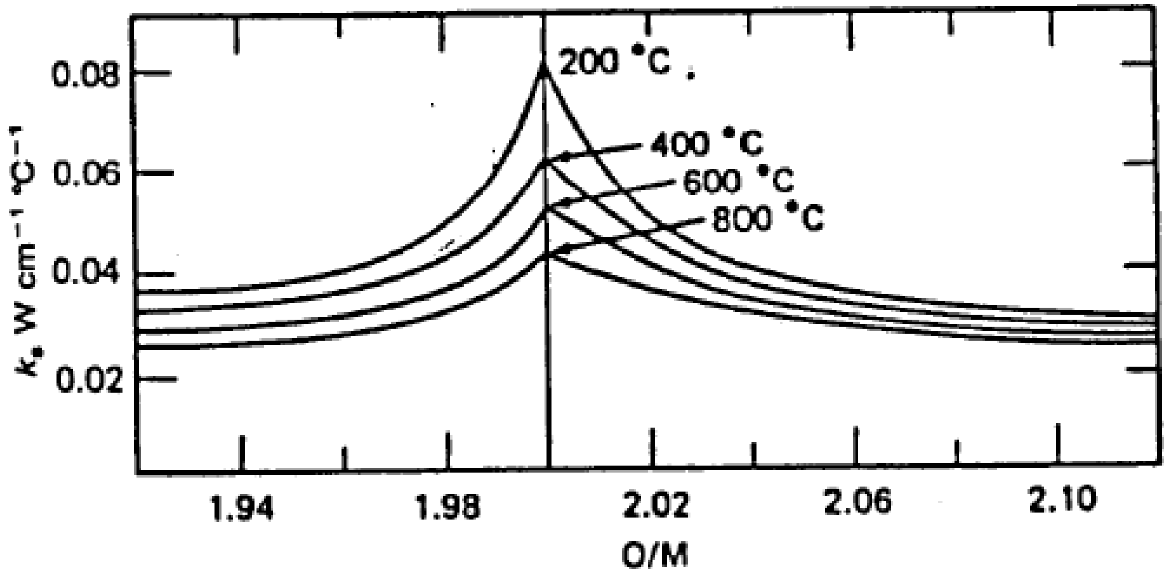


Figure 8-3 Thermal conductivity of $UO_{0.8}Pu_{0.2}O_{2\pm x}$ as a function of the O/(U + Pu) ratio. *(From Schmidt and Richter* [24].*)*

Conduction Heat Transfer Coefficient

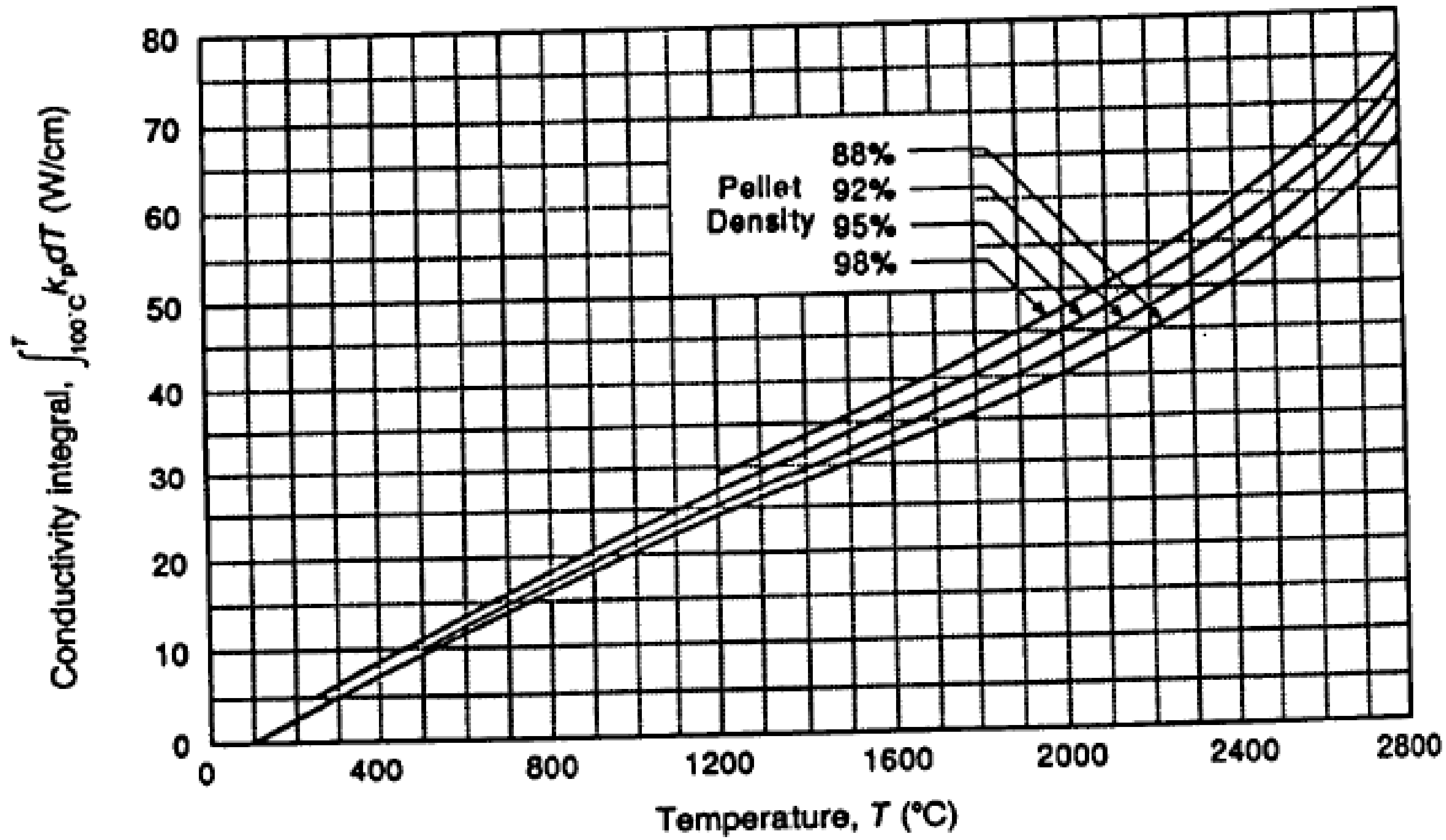
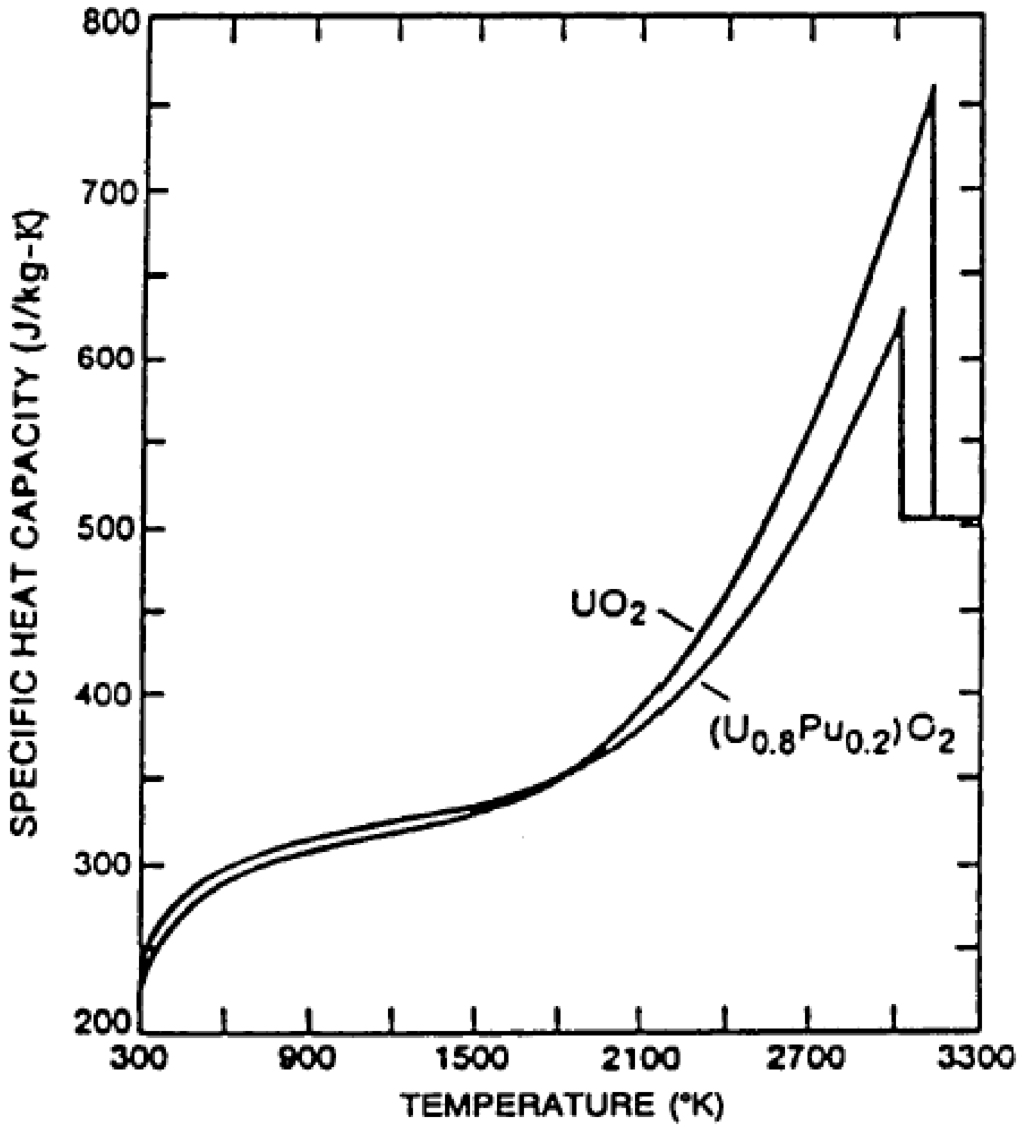
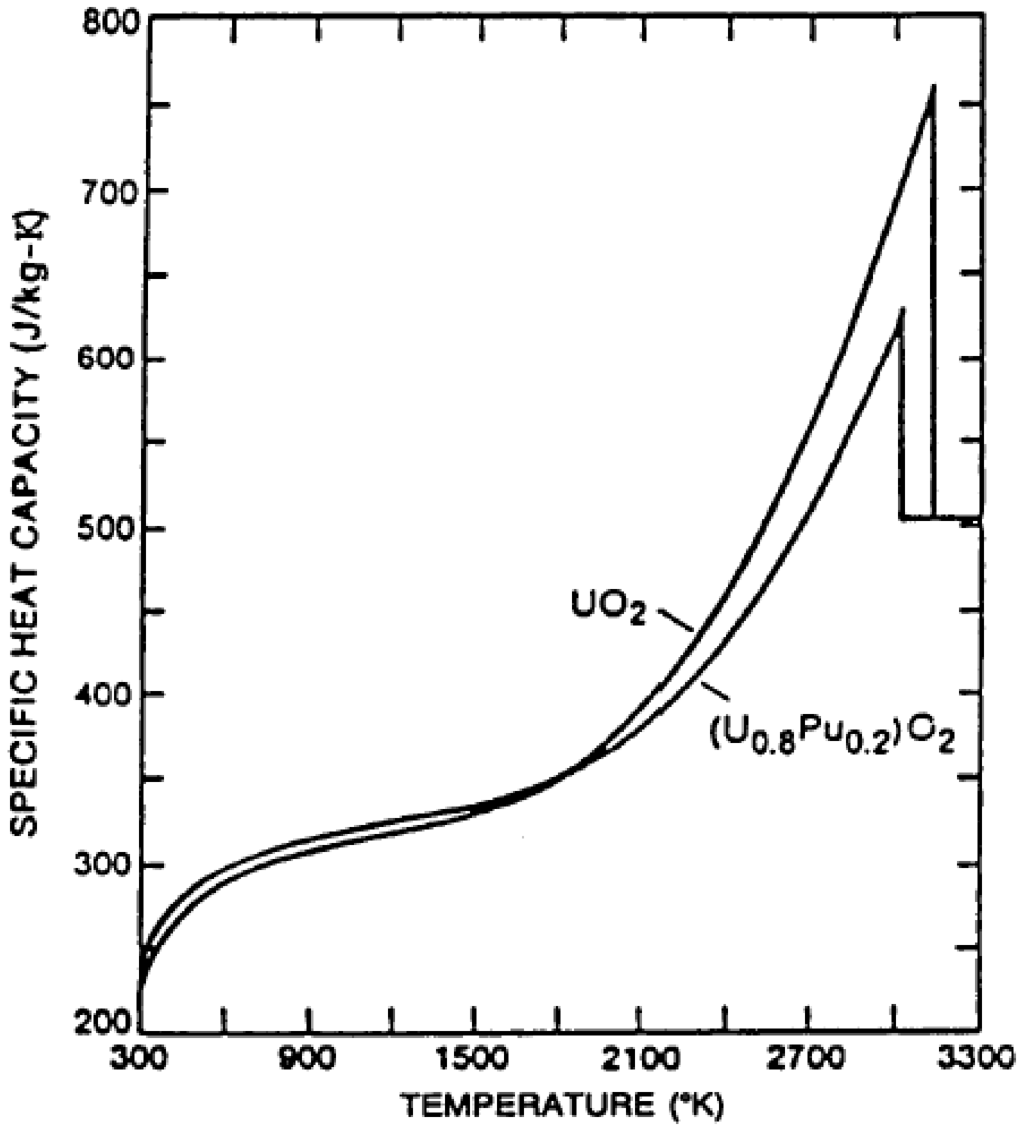


Figure 8-2 $\int_{100^\circ C}^{T} k_p dT$ versus temperature for mixed oxide fuel. Note: 32.8 W/cm = 1 kW/ft.

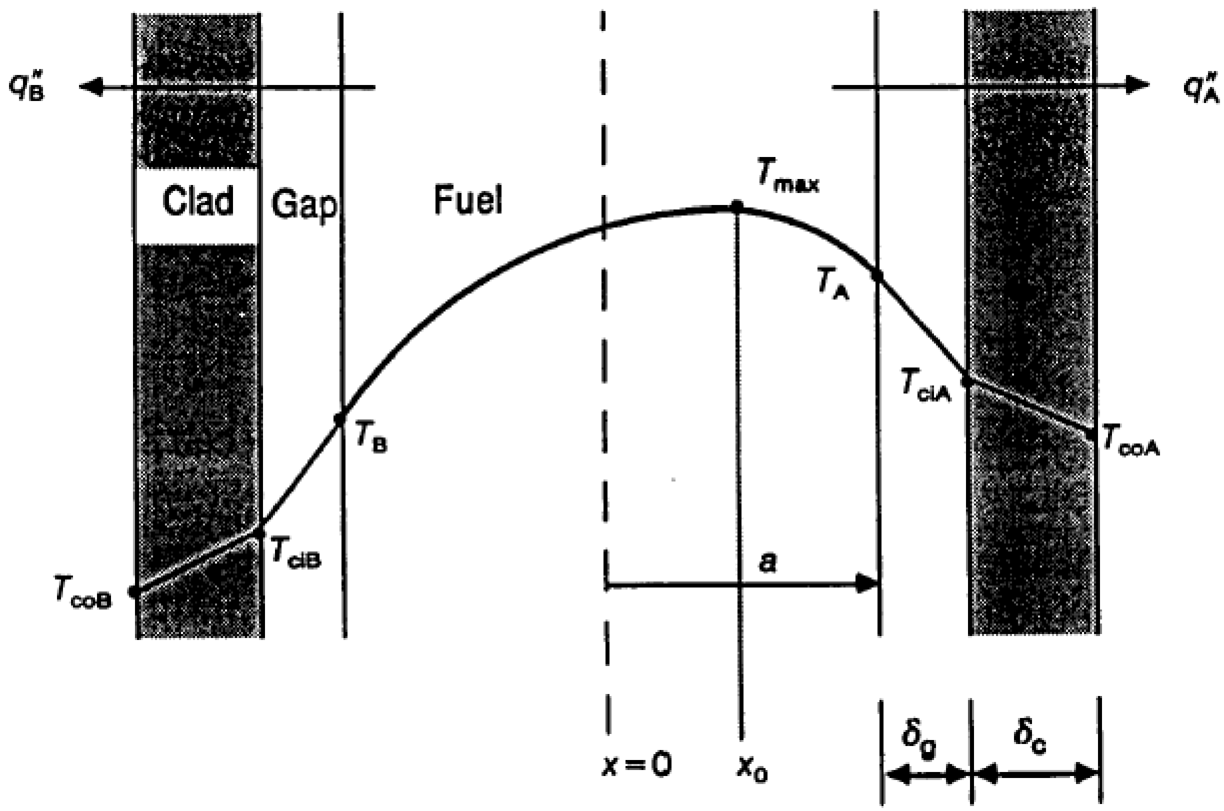
Conduction Heat Transfer Coefficient



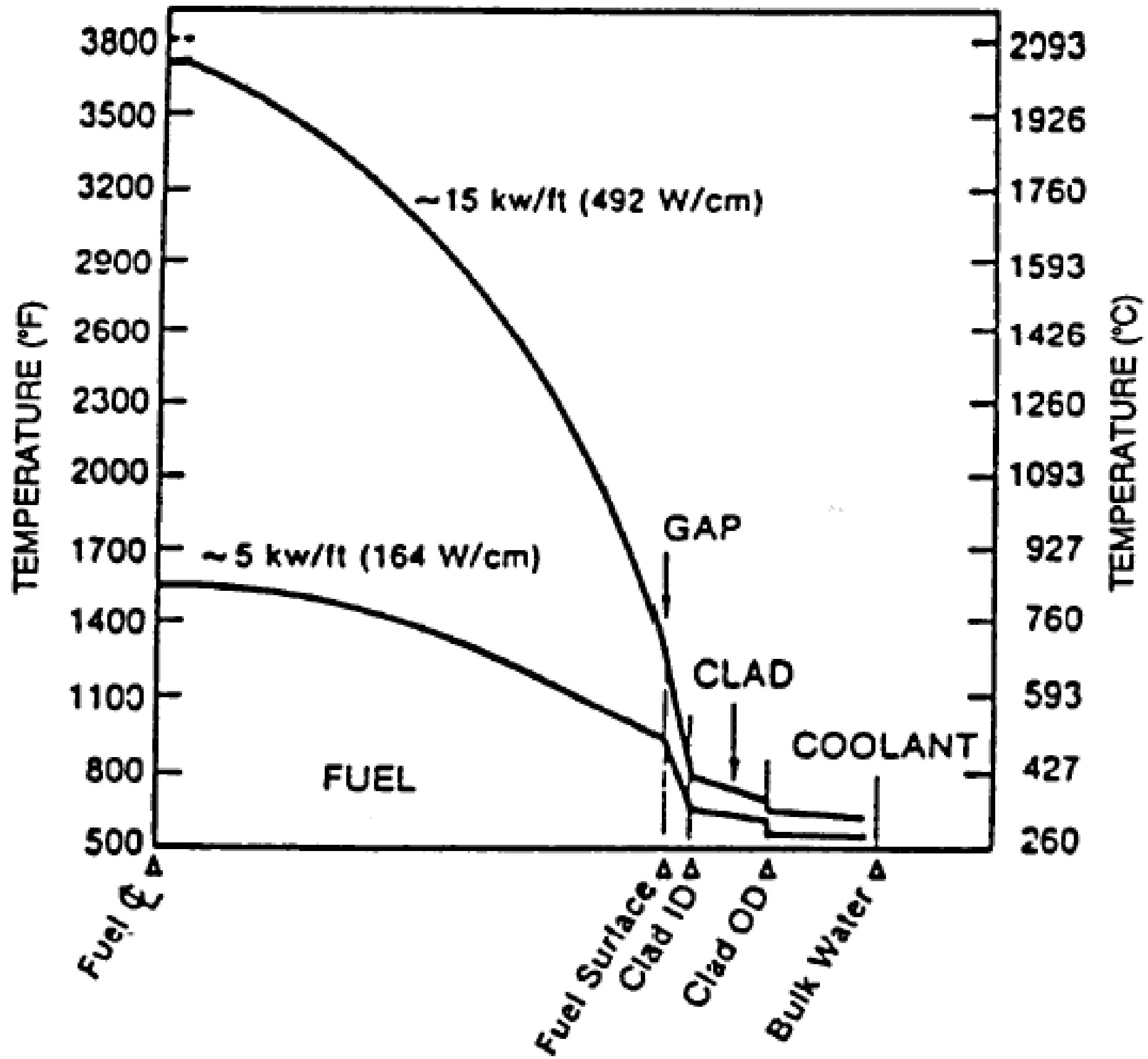
Conduction Heat Transfer Coefficient



Temperature Profiles



Temperature Profiles



Temperature Profiles

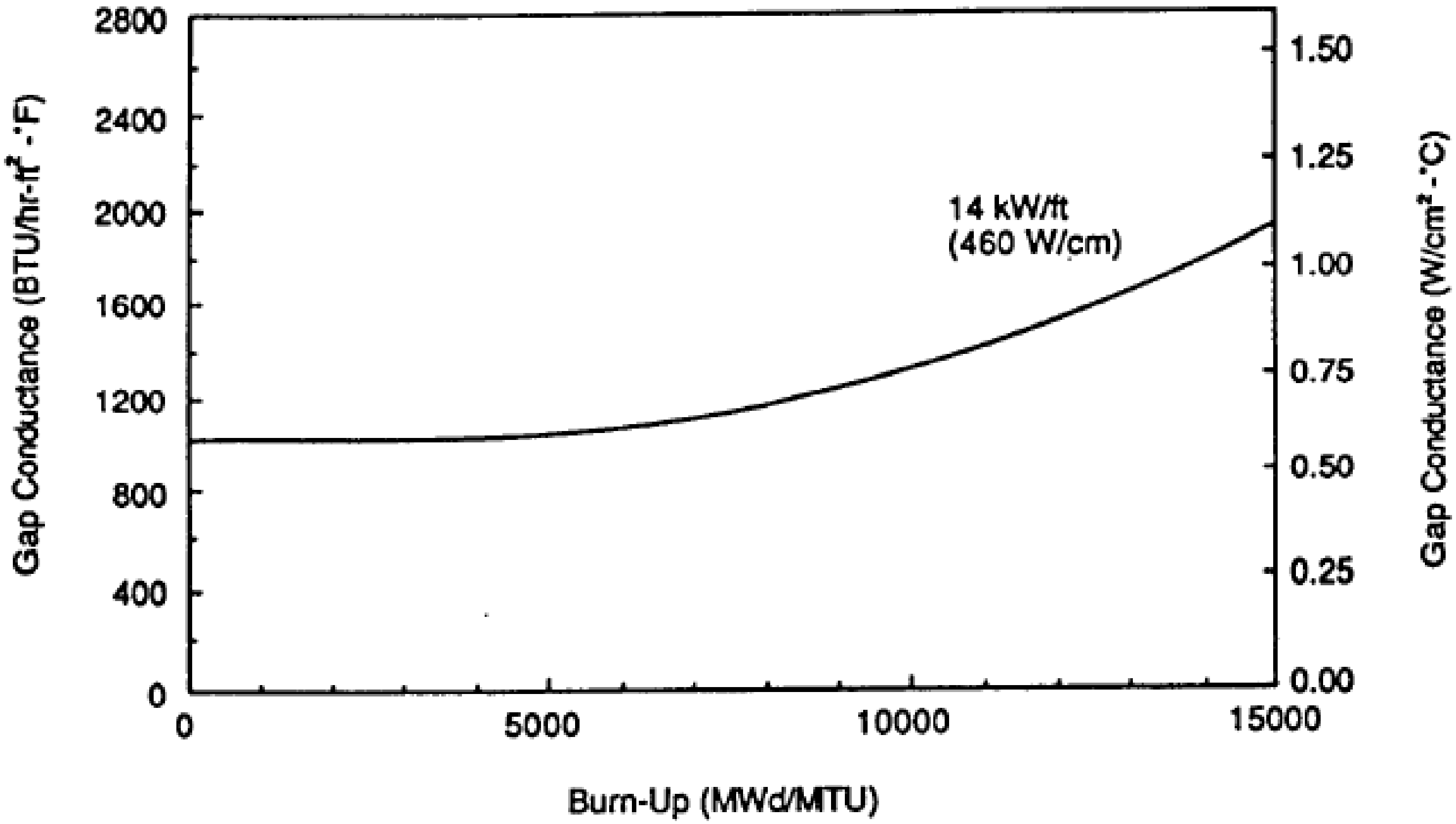
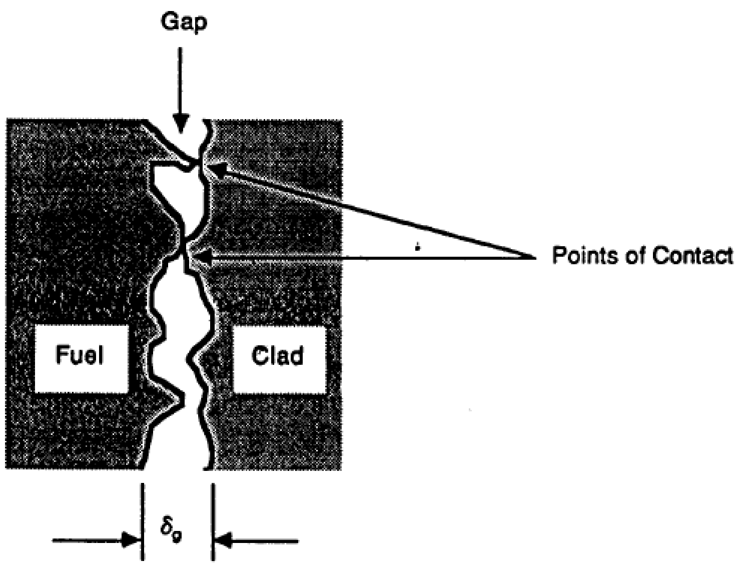


Figure 8-19 Variations of gap conductance with burnup for a PWR fuel rod (pressurized with helium) and operating at 14 kW/ft (460 W/cm). *(From Fenech* [6].*)*

Temperature Profiles



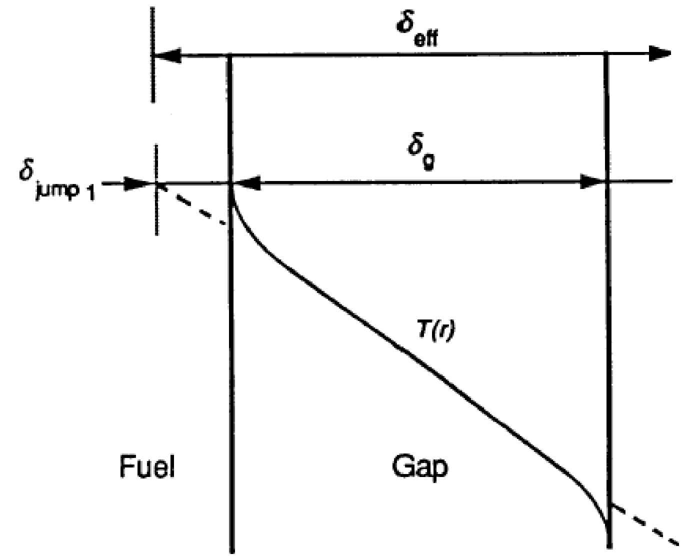


Figure 8-20 Temperature profile across the fuel–clad gap.

Temperature Profiles

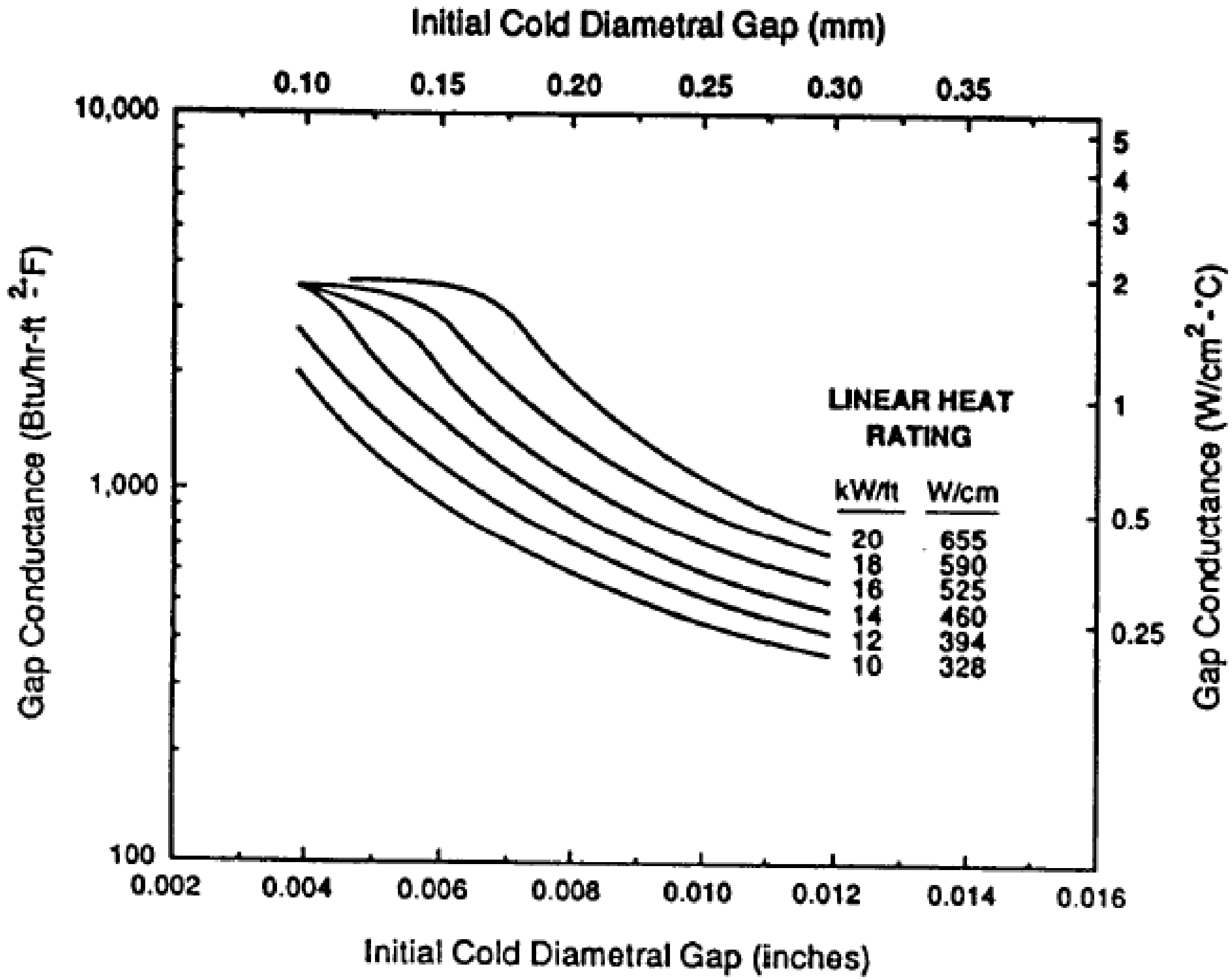


Figure 8-22 Calculated gap conductance as a function of cold diametral gap in a typical LWR fuel rod. *(From Horn and Panisko, [10].)*

Questions?