

UNENE Graduate Course
Reactor Thermal-Hydraulics Design and
Analysis

McMaster University

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Two-Fluid Basic Equations

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Outline

- Two-Fluid Approach
- Principle of conservation
- Conservation of Mass
- Conservation of Momentum
- Conservation of Energy
- Overview of models

Two-Fluid Model Approach

Integral
(Macro)

Distributed
(Micro)

$$\frac{d}{dt} \int_V \delta_k \rho_k dV = - \int_S \delta_k \rho_k \underline{v}_k \cdot \underline{n} ds + \int_V \Gamma_k dV$$

$$\frac{d}{dt} \int_V \delta_k \rho_k \underline{v}_k dV = \int_S \underline{\sigma}_k \cdot \underline{n} ds + \int_V \delta_k \rho_k \underline{f}_k dV + \int_V \underline{m}_k dV$$

$$\frac{d}{dt} \int_V \delta_k \rho_k \left(e_k + \frac{1}{2} v_k^2 \right) dV = - \int_S q_k \cdot \underline{n} ds + \dots$$

Integrate over surfaces
(assume constant properties
over surface)
to get macroscopic
or lumped approach

Apply Gauss' Theorem
 $\int_S \underline{\sigma} \cdot \underline{n} ds = \int_V \nabla \cdot \underline{\sigma} dV$
and drop \int_V
to get microscopic
or differential form

$$\frac{dM}{dt} = \sum W$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \underline{v} = 0$$

SAPHY → Hind's transmission model

COBR → THIRST → MOC3D

Khan's formalism
NUCIRC

**CATHENA
(TUF)**

increased sophistication
↑
Simultaneous solution of
coupled equations
↓
Representations
are equivalent

Integration or
Simultaneous solution
of individual microscopic
equations

Principle of Conservation

- What is conservation of a field variable
 - “*What goes in must come out unless it stays there or is generated or lost somehow*”
- Conservation is applied to mass, momentum and energy (heat) transfer
- Definition of derivatives
 - Partial time derivative ($\partial/\partial t$)
 - Observer fixed in space, partial with respect to time, holding x, y, z constant
 - Total time derivative (d/dt)
 - Observer moving independently; must take into account observer velocity ($d/dt = \partial/\partial t + \partial/\partial x \cdot \partial x/\partial t$)
 - Substantial derivative
 - Derivative following the motion; observer moving with the “fluid” ($D/Dt = \partial/\partial t + \mathbf{v}_x \cdot \partial/\partial \mathbf{x}$)

Conservation Equations

$$\frac{D}{Dt} \iiint \psi \cdot dV = \iiint \Gamma \cdot dV + \iint \vec{S} \cdot \vec{n} \cdot ds$$

Substantial
Derivative

Volume Source

Surface Source

Integral (macroscopic)
form of conservation
equations

$$\frac{D}{Dt} \iiint \psi \cdot dV = \iiint \frac{\partial \psi}{\partial t} \cdot dV + \iint \psi \cdot \vec{v} \cdot \vec{n} \cdot ds$$

$$\iiint \frac{\partial \psi}{\partial t} \cdot dV = \iint \psi \cdot \vec{v} \cdot \vec{n} \cdot ds + \iiint \Gamma \cdot dV + \iint \vec{S} \cdot \vec{n} \cdot ds$$

Gauss' Divergence Theorem

$$\iint \vec{A} \cdot \vec{n} \cdot ds = \iiint \nabla \cdot A \cdot dV$$

$$\iiint \frac{\partial \psi}{\partial t} \cdot dV = -\iiint \nabla \cdot \psi \mathbf{v} \cdot dV + \iiint \Gamma \cdot dV + \iiint \nabla \cdot S \cdot dV$$

Conservation Equations

$$\frac{\partial \psi}{\partial t} = -\nabla \cdot \psi \cdot \nu + \Gamma + \nabla \cdot S$$

Distributed (microscopic)
form of conservation
equations

Storage Flux Volume Source Surface Source

- Conservation is a direct result of experimental observation
- The field variables are continuous within volume V (in time and space)
- Spatial and temporal averaging is used to smooth out variations that are not important to consider

Mass Conservation Equation (Micro)

$$\iiint \frac{\partial}{\partial t} (\gamma_k \cdot \rho_k) dV = - \iint \gamma_k \cdot \rho_k \cdot \vec{v}_k \cdot \vec{n} \cdot ds + \iiint \Gamma_k \cdot dV + \iint \vec{S}_k \cdot \vec{n} \cdot ds$$

$$\begin{aligned} \iiint \frac{\partial}{\partial t} [(1-\alpha)\rho_1 + \alpha\rho_2] \cdot dV &= - \iint [(1-\alpha)\rho_1 \cdot \vec{v}_1 + \alpha\rho_2 \cdot \vec{v}_2] \cdot \vec{n} \cdot ds \\ + \iiint (\Gamma_1 + \Gamma_2) \cdot dV &+ \iint (\vec{S}_1 + \vec{S}_2) \cdot \vec{n} \cdot ds \end{aligned}$$

$$\iiint \frac{\partial}{\partial t} dV = - \iint \rho \cdot \vec{v} \cdot \vec{n} \cdot ds$$

Gauss' Divergence Theorem

$$\iint \vec{A} \cdot \vec{n} \cdot ds = \iiint \nabla \cdot \vec{A} \cdot dV$$

$$\frac{\partial}{\partial t} [(1-\alpha) \cdot \rho_1 + \alpha \cdot \rho_2] + \nabla [(1-\alpha) \cdot \rho_1 \cdot v_1 + \alpha \cdot \rho_2 \cdot v_2] = 0$$

Mass Conservation Equation (Macro)

$$-\iint \rho \cdot \vec{v} \cdot \vec{n} \cdot ds = \sum_j \rho_j \cdot \mathbf{v}_j \cdot A_j$$

$$\frac{\partial M_i}{\partial t} = \sum_j \rho_j \cdot \mathbf{v}_j \cdot A_j = \sum W_j$$

$$\sum_j \rho_j \cdot \mathbf{v}_j \cdot A_j = \sum W_j = 0$$

$$\mathbf{v} = \frac{W}{\rho \cdot A}$$

Momentum Conservation Equation (Micro)

$$\frac{D}{Dt} \iiint \gamma_k \cdot \rho_k \cdot \vec{v}_k \cdot dV = \iint \vec{\sigma}_k \cdot \vec{n} \cdot ds + \iiint \gamma_k \cdot \rho_k \cdot \vec{f}_k \cdot dV + \iiint \vec{M}_k \cdot dV$$

$$\iiint \frac{\partial}{\partial t} \cdot \rho \cdot \vec{v} \cdot dV + \iint \rho \cdot \vec{v} \cdot (\vec{v} \cdot \vec{n}) \cdot ds = \iint \vec{\sigma} \cdot \vec{n} \cdot ds + \iiint \rho \cdot \vec{f} \cdot dV$$

$$\frac{\partial}{\partial t} (\rho \cdot \vec{v}) + \nabla \cdot \rho \cdot \vec{v} \cdot \vec{v} = \nabla \cdot \vec{\sigma} + \rho \cdot \vec{f}$$

$$\frac{\partial}{\partial t} (\rho \cdot \vec{v}) + \nabla \cdot \rho \cdot \vec{v} \cdot \vec{v} = -\nabla \cdot P + \nabla \cdot \vec{\tau} + \rho \cdot \vec{f}$$

Momentum Conservation Equation (Macro)

$$V \cdot \frac{\partial \rho \vec{v}}{\partial t} - A_{IN} \cdot \rho_{IN} \cdot v_{IN} \cdot \vec{v}_{IN} + A_{OUT} \cdot \rho_{OUT} \cdot \vec{v}_{OUT} \cdot v_{OUT} = \iint \vec{\sigma}_k \cdot \vec{n} \cdot ds + \iiint \rho \cdot \vec{f} \cdot dV$$

$$V \cdot \frac{\partial \rho \vec{v}}{\partial t} = - \iint P \cdot \vec{I} \cdot \vec{n} \cdot ds + \iiint (\nabla \cdot \vec{\tau} + \rho \cdot \vec{f}) \cdot dV$$

$$= -A_{OUT} \cdot \vec{P}_{OUT} + A_{IN} \cdot \vec{P}_{IN} - \frac{V \cdot \rho}{L} \left(\frac{f \cdot L}{D} + k \right) \cdot \frac{\vec{v} \cdot |\vec{v}|}{2g_c} - L \cdot A \cdot \rho \cdot \sin(\theta) \cdot \left(\frac{\vec{g}}{g_c} \right)$$

$$\frac{\partial W}{\partial t} = \frac{A}{L} \left[(P_{IN} - P_{OUT}) - \left(\frac{f \cdot L}{D} + k \right) \frac{W^2}{2g_c \cdot \rho \cdot A^2} \right] - A \cdot \rho \cdot \frac{g}{g_c} \sin(\theta)$$

$$P_{IN} - P_{OUT} = \rho \cdot \left(\frac{f \cdot L}{D} + k \right) \cdot \frac{V^2}{2g_c} = \left(\frac{f \cdot L}{D} + k \right) \cdot \frac{W^2}{2A^2 \cdot \rho \cdot g_c} + \Delta P_{PUMP} + \dots$$

Energy Conservation Equation (Micro)

$$\begin{aligned} & \frac{D}{Dt} \iiint \gamma_k \cdot \rho_k \cdot \left(c_k + \frac{1}{2} v_k^2 \right) \cdot dV \\ &= - \iint \vec{q}_k \cdot \vec{n} \cdot ds + \iiint E_k \cdot dV + \iiint \gamma_k \cdot \rho_k \cdot \vec{f}_k \cdot \vec{v}_k \cdot dV + \iint (\vec{\sigma}_k \cdot \vec{n}) \cdot v_k \cdot ds \end{aligned}$$

$$\begin{aligned} & \iiint \frac{\partial}{\partial t} \left[\rho \cdot e + \frac{1}{2} \rho v^2 \right] \cdot dV + \iint \left[\rho \cdot e + \frac{1}{2} \rho v^2 \right] \cdot \vec{v} \cdot \vec{n} \cdot ds = \\ &= - \iint \vec{q} \cdot \vec{n} \cdot ds + \iiint E \cdot dV + \iiint \rho \cdot \vec{f} \cdot \vec{v} \cdot dV + \iint (\vec{\sigma} \cdot \vec{v}) \cdot \vec{v} \cdot ds \end{aligned}$$

$$\begin{aligned} & \iiint \frac{\partial}{\partial t} \left[\rho \cdot e + \frac{1}{2} \rho v^2 \right] \cdot dV + \iint \rho \cdot e \cdot \vec{v} \cdot \vec{n} \cdot ds = \iiint \nabla \cdot \left[\frac{1}{2} \rho \cdot v^2 \cdot \vec{v} \right] \cdot dV \\ &= - \iint \vec{q} \cdot \vec{n} \cdot ds + \iiint E \cdot dV + \iiint \rho \cdot \vec{f} \cdot \vec{v} \cdot dV + \iiint \nabla \cdot (\vec{\sigma} \cdot \vec{v}) \cdot dV \end{aligned}$$

Energy Conservation Equation (Micro)

$$\iiint \frac{\partial}{\partial t} (\rho \cdot \vec{v}) \cdot \vec{v} \cdot dV + \iiint \vec{v} \cdot (\nabla \cdot \rho \cdot \vec{v} \cdot \vec{v}) \cdot dV = \iiint \vec{v} \cdot (\nabla \cdot \vec{\tau}) \cdot dV \\ - \iiint \vec{v} \cdot \nabla P \cdot dV + \iiint \rho \cdot \vec{f} \cdot \vec{v} \cdot dV$$

$$\iiint \frac{\partial}{\partial t} (\rho \cdot e) \cdot dV + \iint \rho \cdot e \cdot \vec{v} \cdot \vec{n} \cdot ds = - \iint \vec{q} \cdot \vec{n} \cdot ds \\ + \iiint E \cdot dV + \iiint \vec{\tau} : \nabla \vec{v} \cdot dV - \iiint P \nabla \cdot \vec{v} \cdot dV$$

$$\frac{\partial}{\partial t} (\rho \cdot e) = -\nabla \cdot q + E$$

$$\rho \cdot C_v \cdot \frac{\partial T}{\partial t} = \nabla \cdot k \cdot \nabla T + E = k \cdot \nabla^2 T + E$$

Energy Conservation Equation (Macro)

$$\iint \rho \cdot e \cdot \vec{v} \cdot \vec{n} \cdot ds = \iint_{A_1} \rho \cdot e \cdot \vec{v} \cdot \vec{n} \cdot ds + \iint_{A_2} \rho \cdot e \cdot \vec{v} \cdot \vec{n} \cdot ds + \dots$$

$$\begin{aligned} \iint \rho \cdot e \cdot \vec{v} \cdot \vec{n} \cdot ds = & - \sum_{INFLOW} \rho \cdot e \cdot \vec{v} \cdot A_i + \sum_{OUTFLOW} \rho \cdot e \cdot \vec{v} \cdot A = - \sum W_{IN} \cdot e_{IN} + \sum W_{OUT} \cdot e_{OUT} \\ & - \iint \vec{q} \cdot \vec{n} \cdot ds + \iiint E \cdot dV = Q \end{aligned}$$

$$\frac{\partial U}{\partial t} = \sum W_{IN} \cdot e_{IN} - \sum W_{OUT} \cdot e_{OUT} + Q + \iiint \vec{\tau} : \nabla \vec{v} \cdot dV - \iiint P \nabla \cdot \vec{v} \cdot dV$$

$$\begin{aligned} \iiint \frac{\partial}{\partial t} (\rho \cdot h) \cdot dV + \iint \rho \cdot h \cdot \vec{v} \cdot \vec{n} \cdot ds = & - \iint \vec{q} \cdot \vec{n} \cdot ds + \iiint E \cdot dV \\ + \iiint \vec{\tau} : \nabla \vec{v} \cdot dV + \iiint \frac{\partial P}{\partial t} \cdot dV + \iint P \cdot \vec{v} \cdot \vec{n} \cdot ds - & \iiint P \nabla \cdot \vec{v} \cdot dV \end{aligned}$$

Energy Conservation Equation (Macro)

$$\frac{\partial H}{\partial t} = + \sum W_{IN} \cdot h_{IN} - \sum W_{OUT} \cdot h_{OUT} + Q + \iiint \vec{\tau} : \nabla \vec{v} \cdot dV + \iiint \left(\frac{\partial P}{\partial t} + \vec{v} \cdot \nabla P \right) \cdot dV$$

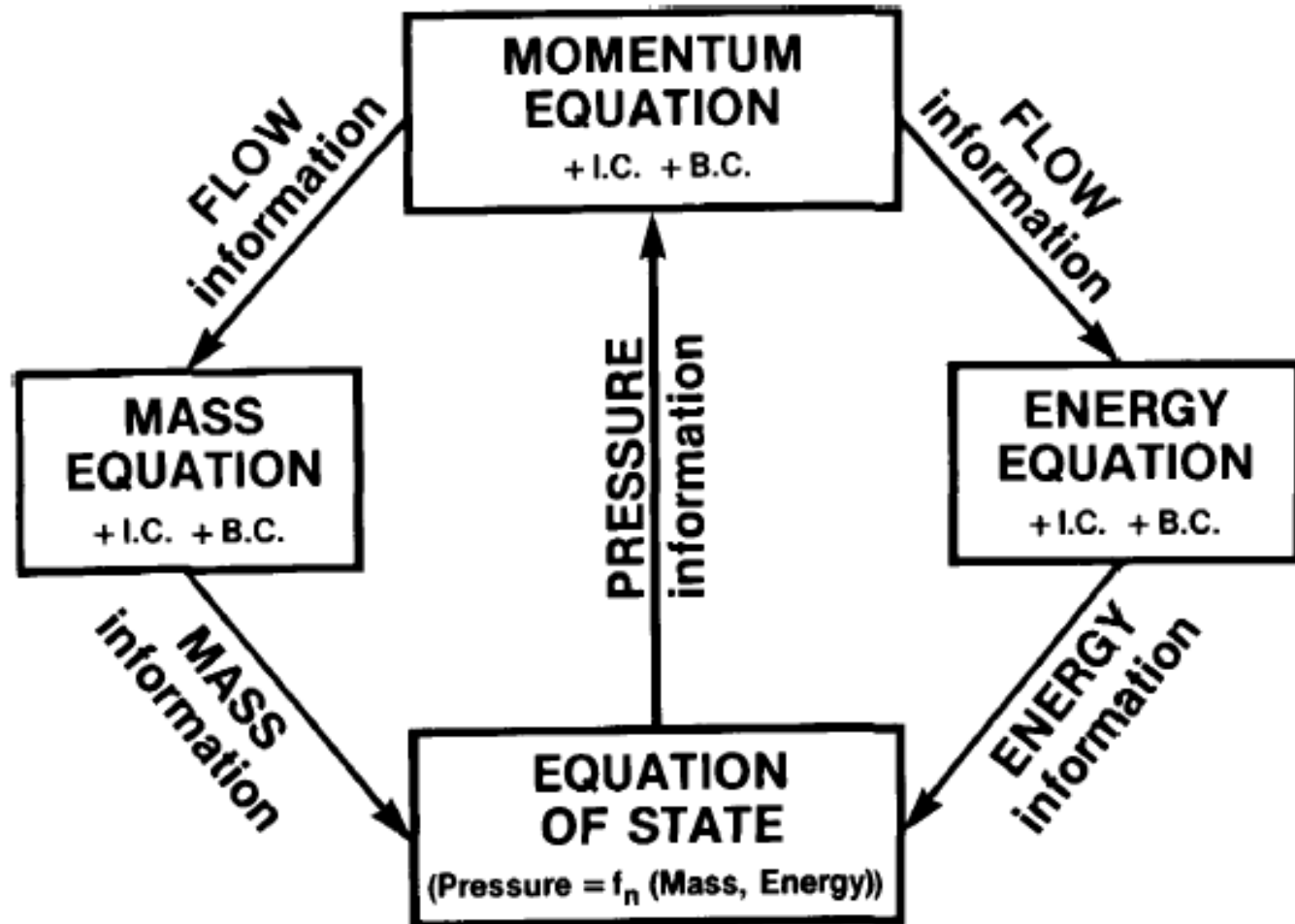
$$Q = \sum W_{OUT} \cdot h_{OUT} - \sum W_{IN} \cdot h_{IN} = W \cdot (h_{OUT} - h_{IN}) = W \cdot C_p \cdot (T_{OUT} - T_{IN})$$

Newton's Law

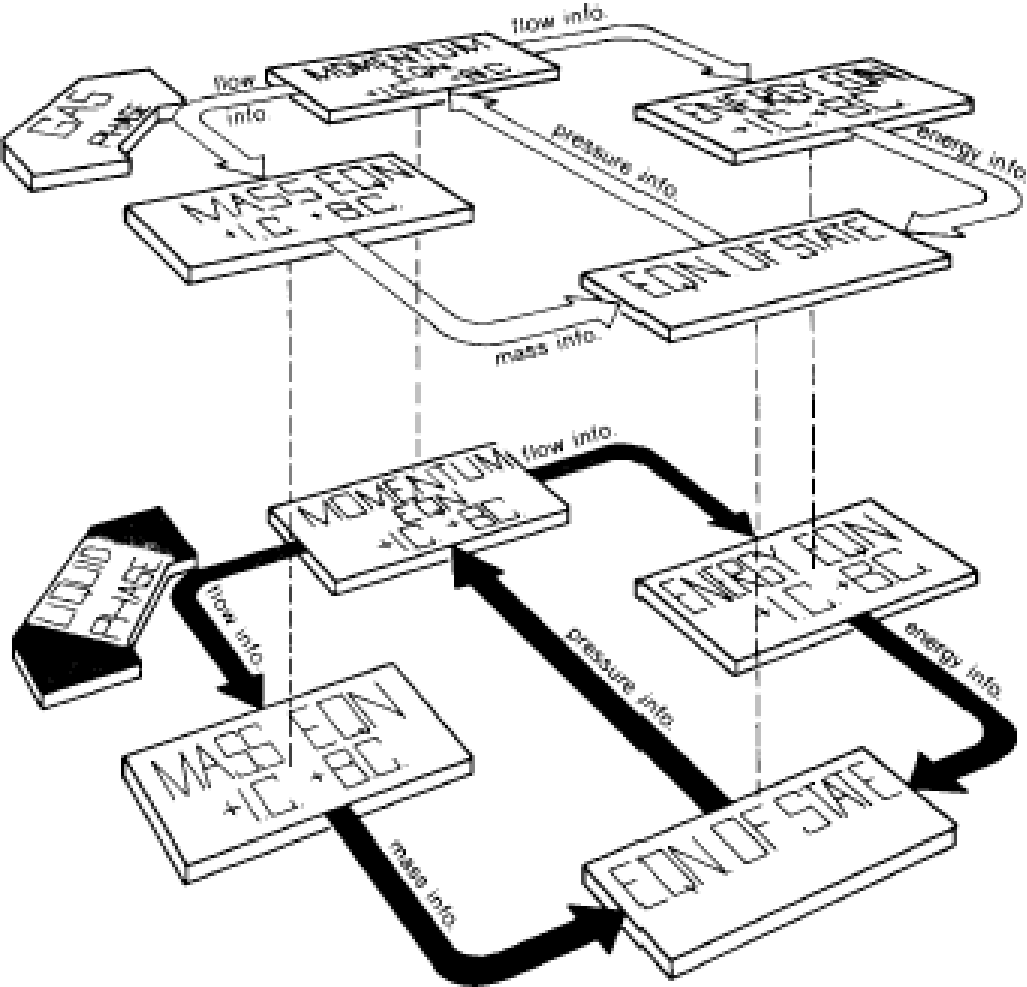
$$\vec{q} \cdot \vec{n} = h_N \cdot (T - T_s)$$

$$V \cdot \frac{\partial \rho h}{\partial t} - V \cdot \frac{\partial P}{\partial t} \left(\equiv V \cdot \frac{\partial \rho e}{\partial t} \approx V \cdot \rho C_v \frac{\partial T}{\partial t} \right) = \sum W_{IN} \cdot h_{IN} - \sum W_{OUT} \cdot h_{OUT} - A \cdot h_N \cdot (T - T_s) + V \cdot E + \iiint \vec{\tau} : \nabla \vec{v} \cdot dV$$

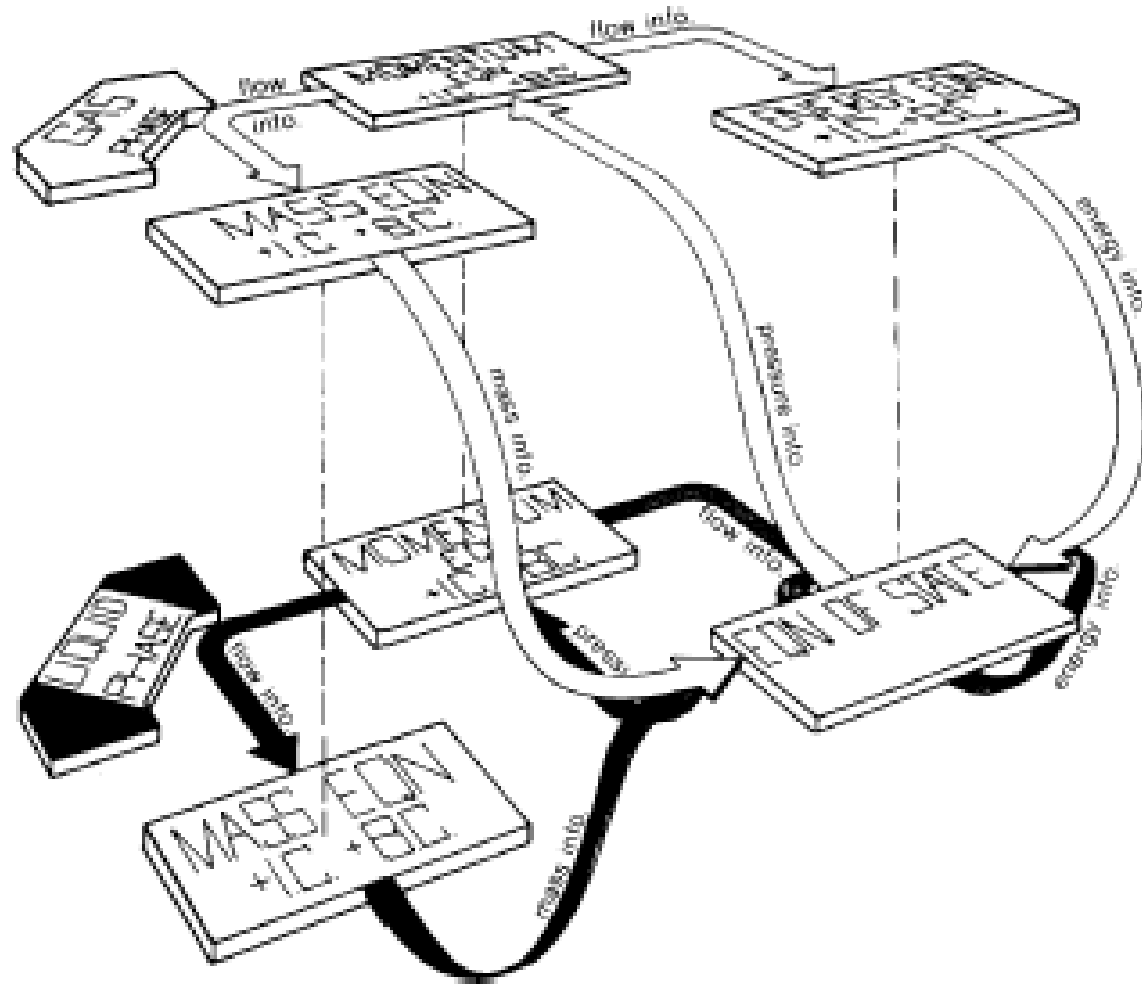
Linkage of Two Fluid Equations



Separate Two Fluid Model



Sombined Two Fluid Model



Questions?