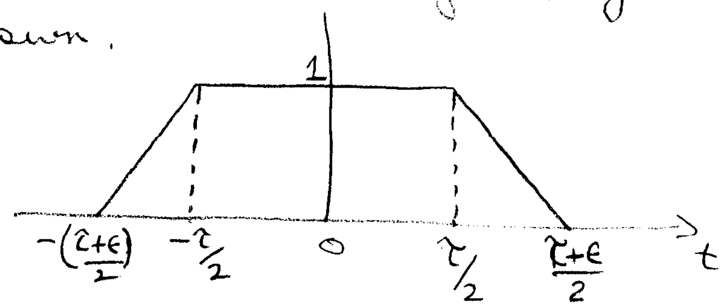
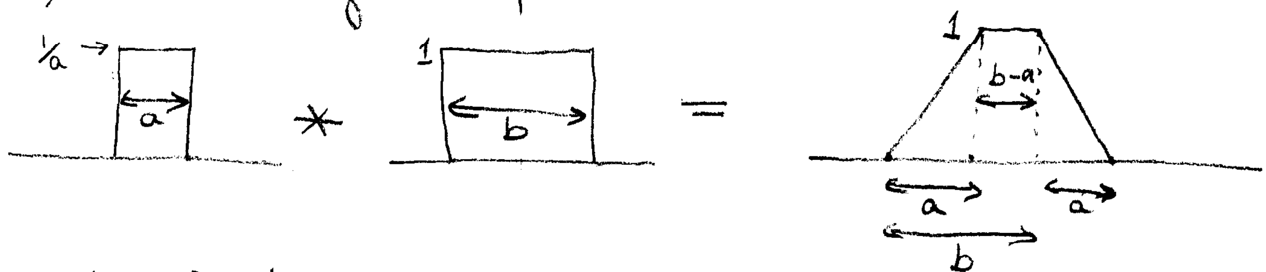
6.1 (a) Calculate the Fourier Transform of the pulse shown.



(b) What is the Fourier Transform as $\epsilon \to \infty$

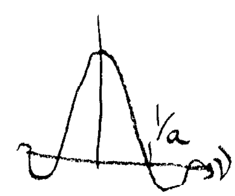
Sol'n

(a) The given pulse is just the convolution of two rectangular pulses:



ie $\tau = b - a$ and $\epsilon/2 = a \Rightarrow b = \tau + \epsilon/2$

We also know ~~that~~ $\Pi_a(t) \rightleftharpoons a \,\text{sinc}(\pi \nu a)$

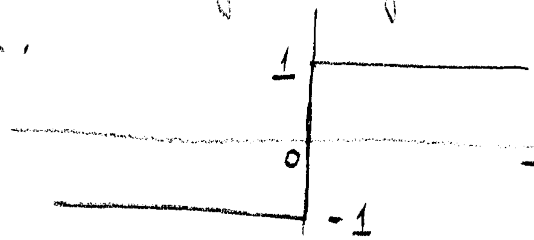


$\therefore \frac{1}{a}\Pi_a(t) * \Pi_b(t) \rightleftharpoons b \,\text{sinc}(\pi \nu a)\,\text{sinc}(\pi \nu b)$

Thus the pulse $\rightleftharpoons$ $\underline{\underline{(\tau + \epsilon/2)\,\text{sinc}\left(\frac{\pi \nu \epsilon}{2}\right)\,\text{sinc}(\pi \nu(\tau + \epsilon/2))}}$

(b) As $\epsilon \to \infty$, the first root of sinc approaches the origin and the height, $2\epsilon(\tau + \epsilon/2)$ increases, ie the F.T. turns into a δ function. Makes sense since the time pulse approaches 1.

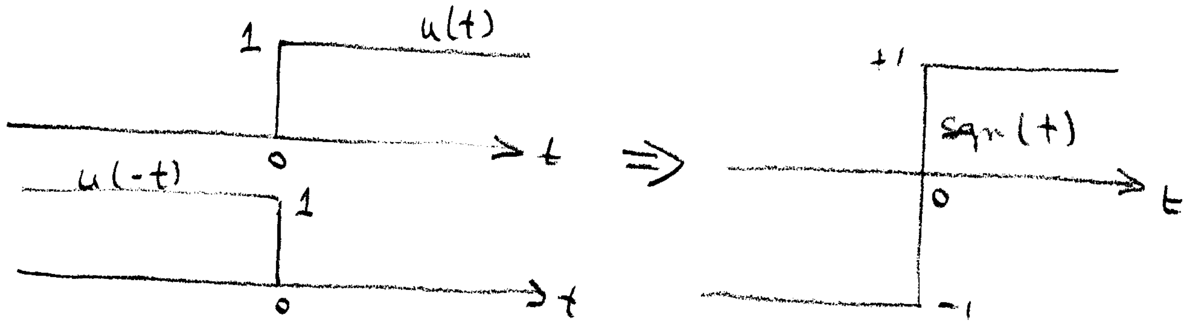
6.2 Calculate the Fourier Transform of the "sgn function" shown.



Hints:
First construct sgn(t):

sgn(t) = u(t) − u(−t) where u(t) is the unit step.
Then find the F.T of u(t) and construct the F.T. of sgn(t) from there.

Soln:



$$u(t) \rightleftharpoons \frac{1}{2\pi i \nu}$$ (This is done in the notes Chapter 8 or look it up in tables).

Since $f(at) \rightleftharpoons \frac{1}{|a|} F(\nu/a)$

then $u(-t) \rightleftharpoons \frac{1}{|-1|} \frac{1}{2\pi i (\nu/(-1))} = -\frac{1}{2\pi i \nu}$

$\therefore \text{sgn}(t) = u(t) - u(-t) = \frac{1}{2\pi i \nu} - \left(-\frac{1}{2\pi i \nu}\right) = \frac{1}{\pi i \nu}$

$$= -\frac{i}{\pi \nu}$$

6.3 The signals $s(t)$ and $g(t)$ are multiplied together and then passed through a linear system, $h(t)$. What is the Fourier Transform of the output signal $w(t)$?

<u>Sol'n</u>

output $= w(t) = [s(t) \cdot g(t)] * h(t)$

$$w(t) \rightleftharpoons W(\nu) = S(\nu) * G(\nu) \cdot H(\nu)$$

$$= \int_{-\infty}^{\infty} S(\nu')\, G(\nu-\nu')\, d\nu' \cdot \int_{-\infty}^{\infty} h(t)\, e^{-2\pi i \nu t}\, dt$$

$$= \left\{ \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} s(t')\, e^{-2\pi i \nu' t'}\, dt' \right) \left(\int_{-\infty}^{\infty} G(t'')\, e^{-2\pi i (\nu-\nu') t''}\, dt'' \right) d\nu' \right\}$$

$$\cdot \int_{-\infty}^{\infty} h(t)\, e^{-2\pi i \nu t}\, dt$$

optional answer.